Bits and Pieces

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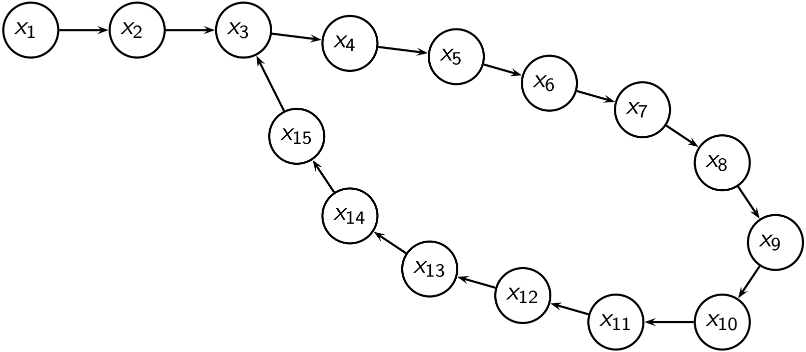
Memoryless Collision Search

- Consider the random function $f : \{0,1\}^n \mapsto \{0,1\}^n$ as a directed graph:
  - Let $V = \{0,1\}^n$ (i.e., each node has a label of length n).
  - and $(x,y) \in E$ if $f(x) = y$.
- A collision in $f(\cdot)$ can be views as two edges (x_1, y) and (x_2, y).

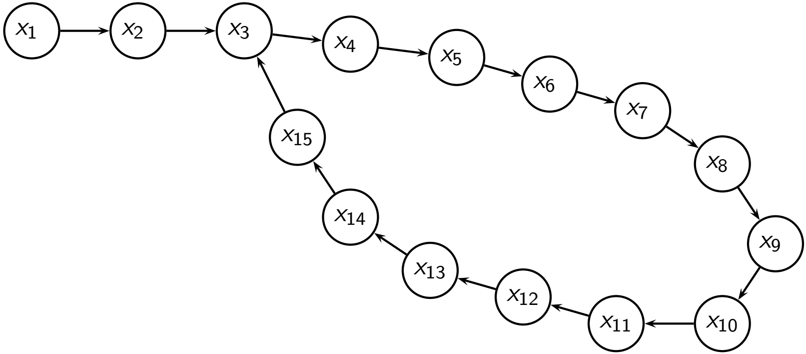
Cycle Finding

- Start from a random node x_1, and compute iteratively $x_{i+1} = f(x_i)$.
- After about $\sqrt{2^n}$ steps, you expect to enter a cycle.
- The entry point (unless it is back to x_1) suggests a cycle.



Floyd's Cycle Finding Algorithm

- Start with two pointers p_1, p_2 initialized both to x_1.
- p_1 is incremented each time by 1 position $p_1 \leftarrow f(p_1)$, and p_2 is incremented each time by 2 positions $p_2 \leftarrow f(f(p_2))$ until they collide.



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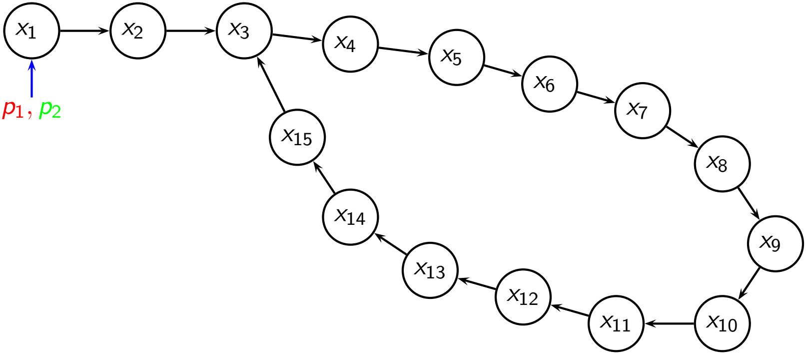

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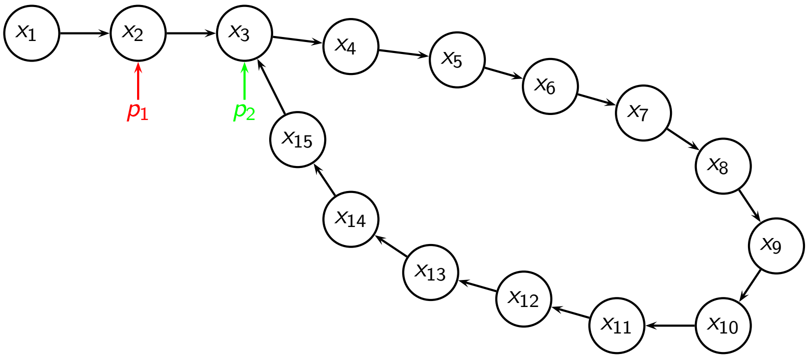

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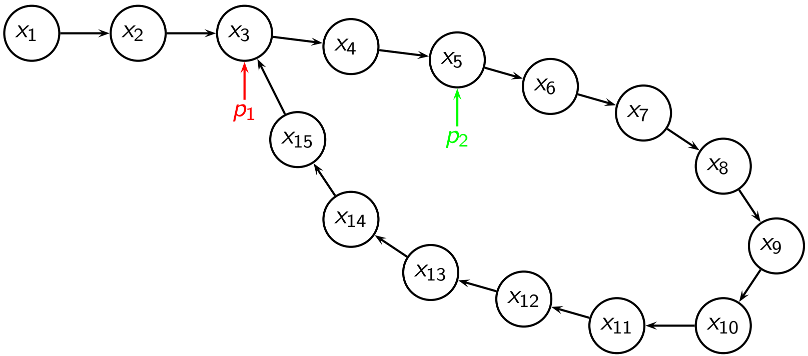

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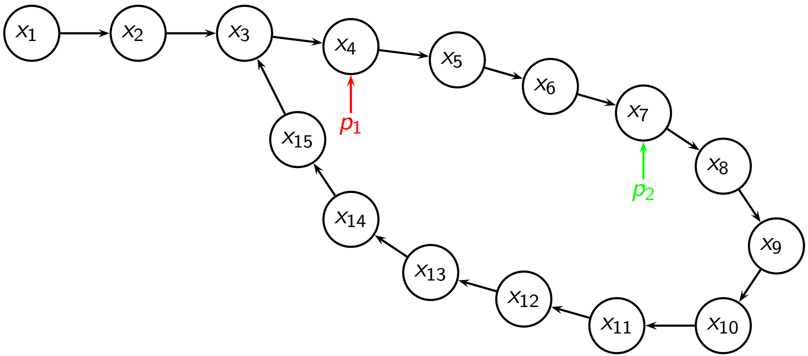

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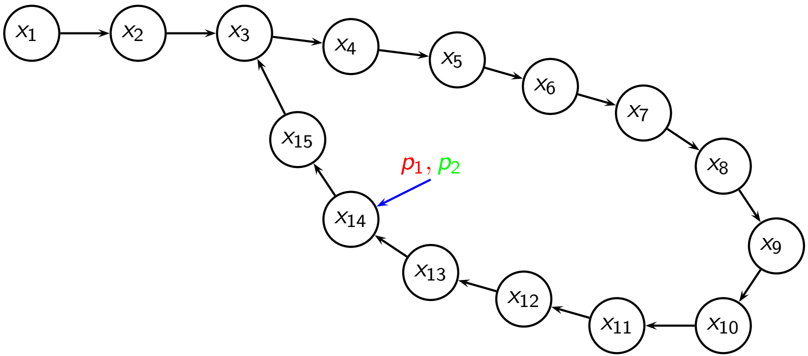

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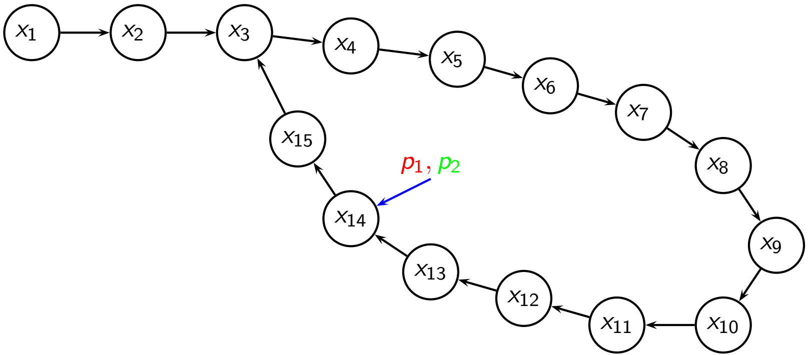

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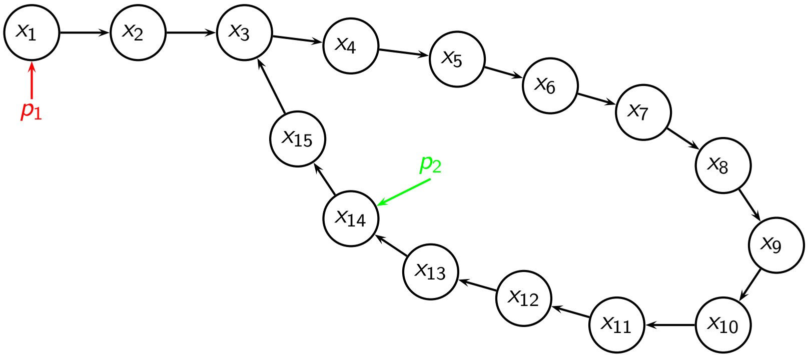

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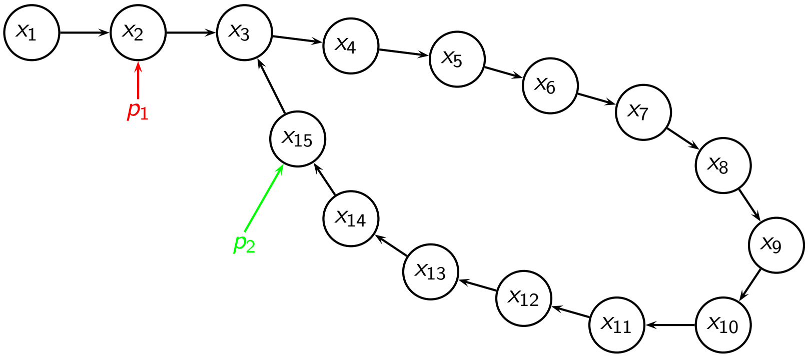

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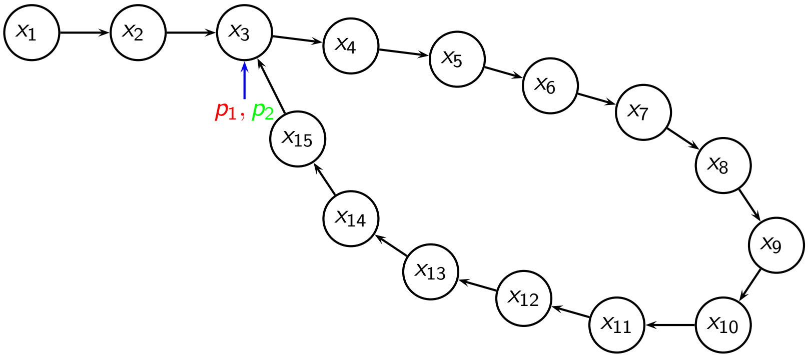


Analysis of Floyd's Cycle Finding Algorithm

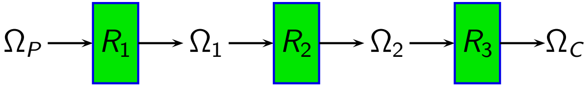
- This method is also known as the ρ-method.
- Let the tail's ($x_1 \rightsquigarrow x_3$) length be ℓ, and let the cycle's length be r. Then if the two pointers collide after t steps:

$$t - \ell = 2t - \ell \bmod r \Rightarrow t \equiv 0 \bmod r$$

- Then, after ℓ more steps, the pointer p_2 is in position $2t + \ell$, which means, it did $2t$ steps inside the cycle, which means that it points to the entry point.
- The algorithm does not work when x_1 is the start of the cycle, or when the cycle is of length 1 (the former is easily solved by picking a different starting point, the latter offers a fixed-point).

Differential Cryptanalysis

- Introduced by Biham and Shamir [BS90].
- Studies the development of differences through the encryption function.
- A differential characteristics $\Omega_P \to \Omega_C$ with probability p:



Performing A Differential Attack

To attack more than r rounds with an r-round differentials:

- Pick T plaintexts which generate $O(1/p)$ pairs of plaintexts with input difference Ω_P.
- Ask for the encryption of these plaintexts.
- Identify among the ciphertexts pairs which may have difference Ω_C.
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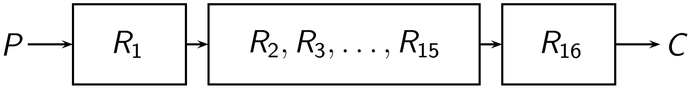

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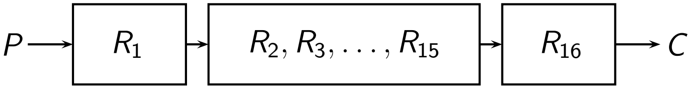

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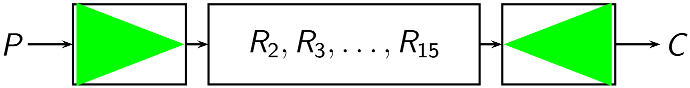

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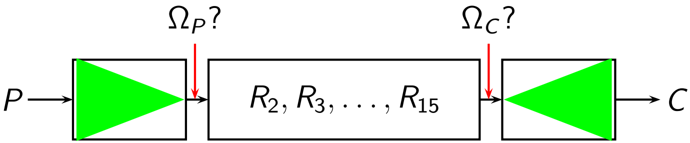

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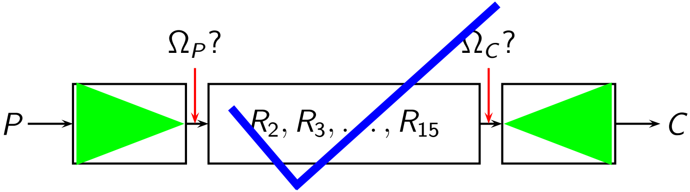

Impossible Differential Cryptanalysis

- Introduced by Biham, Biryukov and Shamir [BBS99].
- Uses differentials with probability **0**.
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- The attack has to discard a large set of (sub)keys, thus it has a lower bound on the time complexity of the attack.

Generic Attack Algorithm

- Let the number of possible subkeys be N_S.
- Pick T plaintexts which generate enough pairs of plaintexts with “input difference” Ω_P and “output difference” Ω_C to discard most of (or all) the $N_S - 1$ wrong subkeys.
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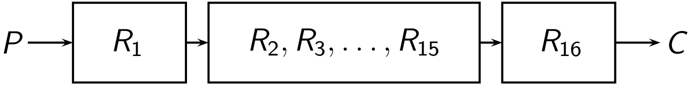

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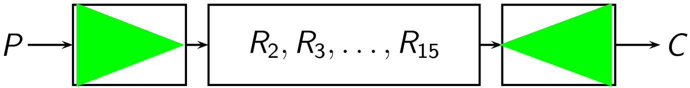

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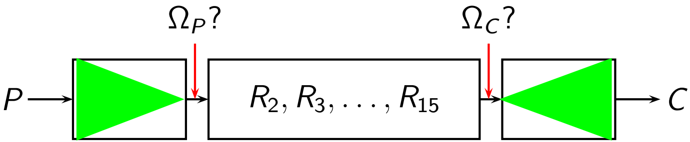

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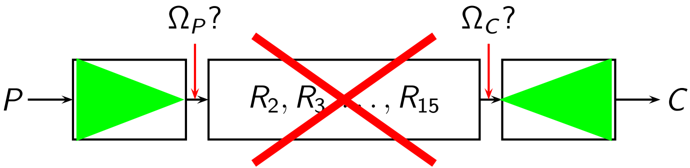

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- and usually they are of little cryptanalytic use.

Miss-in-the-Middle Approach

- The Miss-in-the-Middle approach is based on taking two probability 1 truncated differentials that cannot exist.
- Consider for example the 5-round impossible differential for any Feistel with bijective round functions $(\alpha, 0) \not\rightarrow (\alpha, 0)$.

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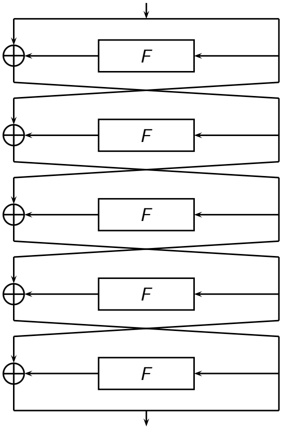

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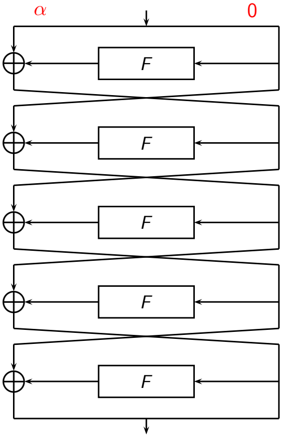

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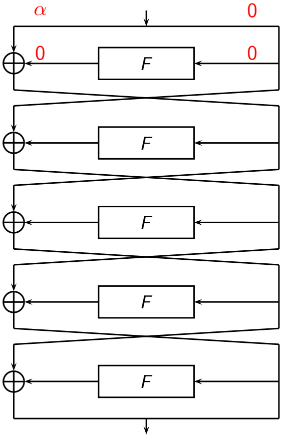

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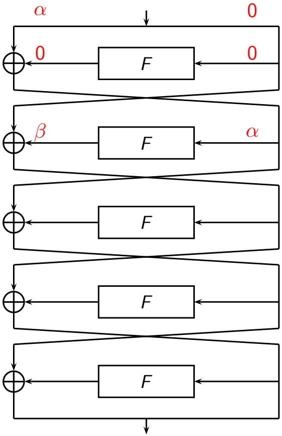

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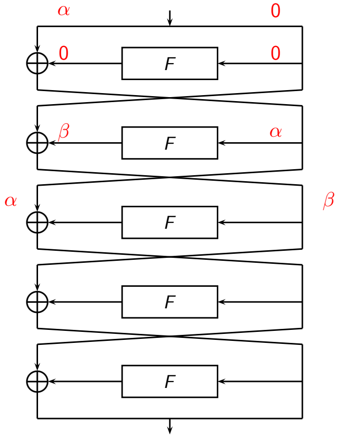

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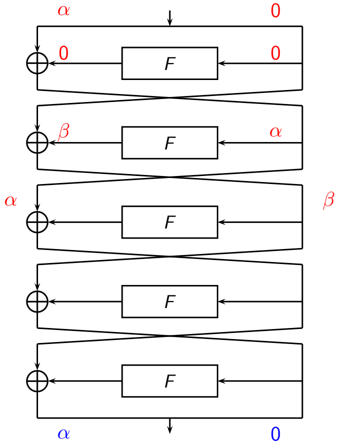

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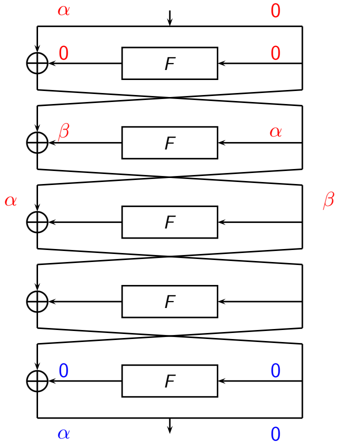

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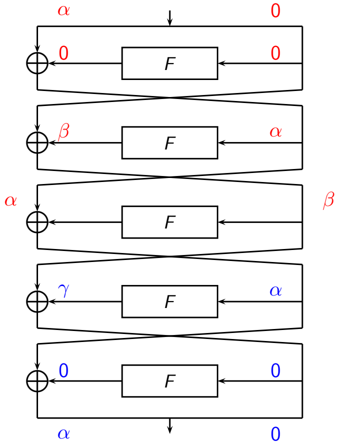

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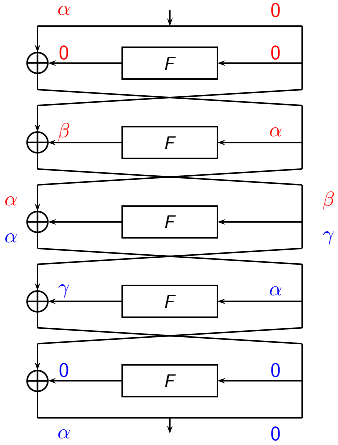

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