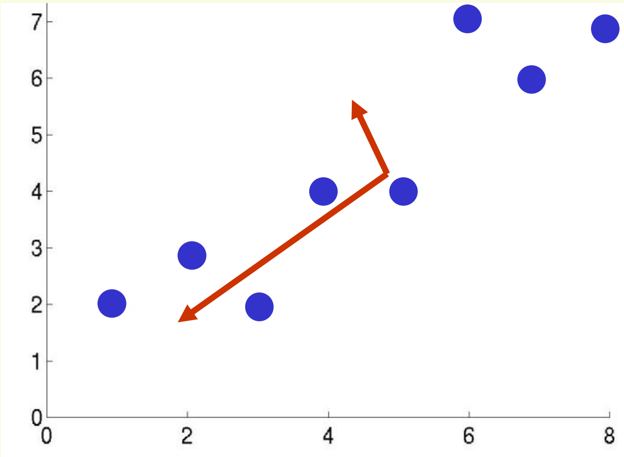
PCA Example Using Matlab

- Let ***D*** = {(1,2),(2,3),(3,2),(4,4),(5,4),(6,7),(7,6),(9,7)}
- Convenient to arrange data in array



$$X = \begin{bmatrix} 1 & 2 \\ \vdots & \vdots \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_8 \end{bmatrix}$$

- Mean $\mu = mean(X) = [4.6 \;\; 4.4]$
- Subtract mean from data to get new data array Z

$$Z = X - \begin{bmatrix} \mu \\ \vdots \\ \mu \end{bmatrix} = X - repmat(\mu,8,1) = \begin{bmatrix} -3.6 & -4.4 \\ \vdots & \vdots \\ 4.4 & 2.6 \end{bmatrix}$$

- Compute the scatter matrix ***S***

$$S = 7 * cov(Z) = [-3.6 \;\; -4.4]\begin{bmatrix} -3.6 \\ -4.4 \end{bmatrix} + \dots + [4.4 \;\; 2.6]\begin{bmatrix} 4.4 \\ 2.6 \end{bmatrix} = \begin{bmatrix} 57 & 40 \\ 40 & 34 \end{bmatrix}$$

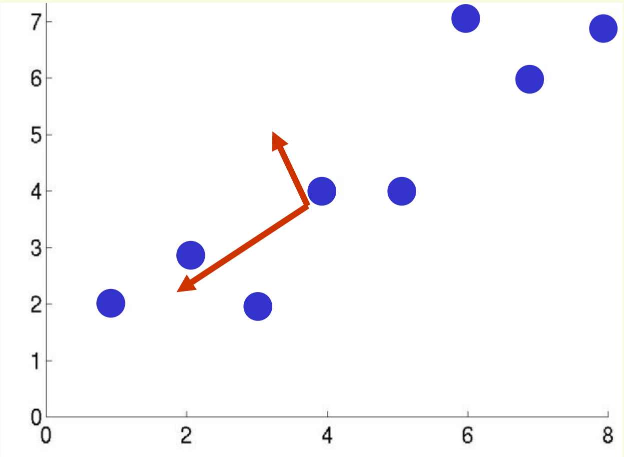
*matlab uses unbiased estimate for covariance, so S=(n-1)*cov(Z)*

PCA Example Using Matlab

- Use [V,D] =eig($\boldsymbol{S}$) to get eigenvalues and eigenvectors of $\boldsymbol{S}$

$$\lambda_1 = 87 \text{ and } e_1 = \begin{bmatrix} -0.8 \\ -0.6 \end{bmatrix}$$

$$\lambda_2 = 3.8 \text{ and } e_2 = \begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix}$$



- Projection to 1D space in the direction of $\boldsymbol{e_1}$

$$Y = e_1^t Z^t = \left(\begin{bmatrix} -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} -3.6 & \cdots & 4.4 \\ -4.4 & \cdots & 2.6 \end{bmatrix} \right) = \begin{bmatrix} 4.3 & \cdots & -5.1 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & \cdots & y_8 \end{bmatrix}$$