Key Recovery Attacks of Practical Complexity on AES Variants

Alex Biryukov, Orr Dunkelman, Nathan Keller, Dmitry Khovratovich, Adi Shamir

Département d'Informatique
École Normale Supérieure

France Telecom Chaire

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Outline

1 AES
   - Specifications
   - The Security of AES

2 Certificational Attacks
   - Historical Overview of Cryptanalysis
   - Current State of Events
   - What a Break is?

3 Attacks on AES-256
   - The Model is All

4 Our Results
   - The Key Point
   - Verification
   - Other Attack Scenarios

5 Summary
The Advanced Encryption Standard

- Designed by Vincent Rijmen and Joan Daemen, under the name Rijndael and submitted to NIST’s competition in 1998.
- Selected after a three year competition as the new standard.
- The cipher has an SP network structure.
- Block size — 128 bits, Key size — 128, 192, or 256 bits.
- Number of rounds depends on the key length (10/12/14, respectively).
The Advanced Encryption Standard
AES’ Key Schedule Algorithm

AES has three key schedules. One for each key size.

- **AES-128** ($Nk = 4$) and **AES-192** ($Nk = 6$):
  1. Initialize $W[0, \ldots, Nk - 1]$ with the user supplied key.
  2. For $i = Nk, \ldots, 43/51$ do
     - If $i \equiv 0 \pmod{Nk}$ then
       $$W[i] = W[i - Nk] \oplus SB(W[i - 1] \ll 8) \oplus RCON[i/Nk],$$
     - Otherwise $W[i] = W[i - 1] \oplus W[i - Nk],$

- **AES-256** ($Nk = 8$):
  1. Initialize $W[0, \ldots, 7]$ with the user supplied key.
  2. For $i = 8, \ldots, 59$ do
     - If $i \equiv 0 \pmod{Nk}$ then
       $$W[i] = W[i - Nk] \oplus SB(W[i - 1] \ll 8) \oplus RCON[i/Nk],$$
     - Else if $i \equiv 4 \pmod{Nk}$ then
       $$W[i] = W[i - 8] \oplus SB(W[i - 1]),$$
     - Otherwise $W[i] = W[i - 1] \oplus W[i - Nk],$
Security Properties

- The S-boxes are based on inversion over $GF(2^8)$.
- The MixColumns operation is an MDS matrix, which along with the ShiftRows operation ensures full diffusion after two rounds.
- The “wide trail strategy” assures that the number of active S-boxes in any differential characteristic is at least five for two rounds, nine for three rounds, and 21 for four rounds.
- There structure offers some 4-round impossible differentials, and several sets of 4-round Square properties.
Differential/Linear Cryptanalysis

- The security against these attacks is derived from the fact that there are no good differentials (linear hulls) of high probability.
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- The security against these attacks is derived from the fact that there are no good differentials (linear hulls) of high probability.
- In a series of papers, the maximal expected differential and linear probabilities for two and four rounds were computed.
- The results are that 4-round AES have no differentials or linear hulls with high enough probability for attacks.
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- 1997 — AES competition. One strike and your out!
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- 1992/3 — related-key attacks (known/chosen key relations).
- 1997 — AES competition. One strike and your out!
- 1999 — Adaptive chosen plaintext and ciphertext attacks (boomerang attacks).
Current State of Affairs in Cryptanalysis

Time complexity of a related-key attack:

“Thus, the total time complexity of Step 2(b) is about $2^{256} \cdot 2^{167.0} = 2^{423.0}$ SHACAL-1 encryptions.”

- Most cryptanalytic papers discuss certificational attacks:
  - Data complexity — just slightly less than the entire code book.
  - Time complexity — just slightly less than exhaustive search.
  - Memory — store more information than there are particles in the universe
Current State of Affairs in Cryptanalysis (cont.)

- These certificational attacks are of great importance:
  1. Why to use a primitive whose less secure than optimal?
Current State of Affairs in Cryptanalysis (cont.)

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But they do not help answering questions by users:

1. Does this attack affect my system?
2. Should I still use AES-256 for encryption?
3. MD5 is still OK for certificates, right?
What a Break is?

- There is an ongoing debate what a broken scheme is.
What a Break is?

- There is an ongoing debate what a broken scheme is. Even from the theoretical point of view.
- The extreme approach: \( \max(\text{Time, Data, Memory}) \) less than Exhaustive search’ time.
- Another approach: \( (\text{Time, Data, Memory}) \) better then generic attacks (time-memory-data tradeoff attacks).
- \( \text{Time} \times \text{Memory} < \) Exhaustive search.
- Money for finding a key in a given time < for a generic attack.
What is a Practical Attack?

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- $2^{55}$ DES encryptions are feasible . . .
- $2^{61}$ SHA-1 evaluations did not complete . . .
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- $2^{55}$ DES encryptions are feasible . . .
- $2^{61}$ SHA-1 evaluations did not complete . . .
- So, let’s take $2^{64}$ cycles
  - which are about $2^{56}$ AES encryptions.
- This is also a restriction on the data complexity.
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Time Complexity of Attacks on AES-256

Exhaustive search

Practical

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 32 64 96 128 160 192 224 256
Time Complexity of Attacks on AES-256

Exhaustive search

Practical
Time Complexity of Attacks on AES-256

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0 32 64 96 128 160 192 224 256
Time Complexity of Attacks on AES-256

Exhaustive search

Practical

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Practical Complexity Attacks on AES
Time Complexity of Attacks on AES-256

Exhaustive search

Practical

$2^{31}$

$2^{39}$
Time Complexity of Attacks on AES-256

Exhaustive search

Practical

Exhaustive search

Practical

\[2^{26.5}\]

\[2^{70}\]

\[2^{45}\]

\[2^{32}\]
The Related-Key Model

- First introduced by Knudsen and Biham, independently.
- The adversary is assumed to have some knowledge on the relation.
- In 1996/7, the concept of related-key differentials was introduced, along with it, the concept where the adversary is allowed to chose the key relation.
- There are “good relations” (XORs, rotations, or additions), and “bad relations” (AND, ORs, XORs + additions together).
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- There are “good relations” (XORs, rotations, or additions), and “bad relations” (AND, ORs, XORs + additions together).
- At the end, the main issue is applicability — does the attack scenario allows this relation or not.
Example: Related-Key Differentials

- The probability of a regular differential is:

\[ \text{Pr}_{P,K}[E_K(P) \oplus E_K(P \oplus \Delta P) = \Delta C] \]

- The probability of a related-key differential is:

\[ \text{Pr}_{P,K}[E_K(P) \oplus E_{K \oplus \Delta K}(P \oplus \Delta P) = \Delta C] \]

- The key difference leads to subkey differences, that may be used to cancel the differences in the input to the round function.
The Related-Subkey Model

- This new model was recently introduced in [BK09].
- In related-key attacks, a simple relation $R$ is used for the keys $K_1, K_2$.
- In related-subkey attacks, $R$ is a simple relation between two subkeys, $RK_1, RK_2$.
- The two subkeys are then handled by the key schedule algorithm to obtain the actual keys.
- This slightly less intuitive approach (and less practical one) can be “covered” by the theoretical treatment by just expanding the set of “good relations”.
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An Interesting Property of the Key Schedule Algorithm of AES-256

Our results are based on the fact that key difference leads to the 10 subkey differences with probability 1!
An 8-Round Related-Key Differential

The probability is $2^{-56}$. It can be transformed into a truncated one predicting 24 bits of difference with probability $2^{-36}$. 
We have verified experimentally the correctness of the 7-round related-key differential derived from the 8-round one (it has probability $2^{-30}$).

We performed 100 experiments, each with a random key and $2^{32}$ random plaintext pairs.

<table>
<thead>
<tr>
<th>Pairs</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>1.8</td>
<td>7.3</td>
<td>14.7</td>
<td>19.5</td>
<td>19.5</td>
<td>15.6</td>
<td>10.4</td>
</tr>
<tr>
<td>Experiment</td>
<td>0</td>
<td>10</td>
<td>18</td>
<td>10</td>
<td>28</td>
<td>18</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pairs</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>6.0</td>
<td>3.0</td>
<td>1.3</td>
<td>0.5</td>
<td>0.2</td>
<td>0.06</td>
</tr>
<tr>
<td>Experiment</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
A 10-Round Related-Subkey Differential

- In the related-subkey model, it is possible to pick two keys which satisfy the difference in a slightly different manner.
- The related-subkey allows for shifting the differential by one round.
- This allows an extension of the differential in the backwards direction (despite having a highly active state).
- Which in turn, allows for attacks of practicaly complexity of up to 10 rounds, and semi-practical of up to 11 rounds.
Other Attack Scenarios

- The attacks work when the plaintexts are generated not randomly as well.
- For example, when counter mode is used. The encryption system is initialized to two initial states and are allowed to generate data sequentially. This simplifies the attack model.
- The attacks are applicable when the plaintexts are ASCII characters (as some key differences are suitable).
- Or even when they are ASCII characters representing only numeric values.
- The minimal hamming weight of the key difference is 24.
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## Summary of the Attacks

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Scenario</th>
<th>Time</th>
<th>Data</th>
<th>Memory</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Key Diff. – CP</td>
<td>$2^{31}$</td>
<td>$2^{31}$</td>
<td>2</td>
<td>Distinguisher</td>
</tr>
<tr>
<td>8</td>
<td>Subkey Diff. – CC</td>
<td>$2^{26.5}$</td>
<td>$2^{26.5}$</td>
<td>$2^{26.5}$</td>
<td>35 subkey bits</td>
</tr>
<tr>
<td>9</td>
<td>Key Diff. – CP</td>
<td>$2^{39}$</td>
<td>$2^{38}$</td>
<td>$2^{32}$</td>
<td>Full key</td>
</tr>
<tr>
<td>9</td>
<td>Subkey Diff. – CC</td>
<td>$2^{32}$</td>
<td>$2^{32}$</td>
<td>$2^{32}$</td>
<td>56 key bits</td>
</tr>
<tr>
<td>10</td>
<td>Subkey Diff. – CP</td>
<td>$2^{49}$</td>
<td>$2^{48}$</td>
<td>$2^{33}$</td>
<td>Distinguisher</td>
</tr>
<tr>
<td>10</td>
<td>Subkey Diff. – CC</td>
<td>$2^{45}$</td>
<td>$2^{44}$</td>
<td>$2^{33}$</td>
<td>35 subkey bits</td>
</tr>
<tr>
<td>11</td>
<td>Subkey Diff. – CP</td>
<td>$2^{70}$</td>
<td>$2^{70}$</td>
<td>$2^{33}$</td>
<td>50 subkey bits</td>
</tr>
</tbody>
</table>

Orr Dunkelman  
Practical Complexity Attacks on AES
Security Implications

- Extending AES-128 key to 256 bits actually reduces security!
- The security margins of AES-256 are smaller than expected.
- The related-subkey model — many new results awaiting.
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- The security margins of AES-256 are smaller than expected.
- The related-subkey model — many new results awaiting.
- This is a good time to check that Serpent-support...
Conclusions

- Did we break the full AES with practical complexity?
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Thank you for your attention!

The paper is available on ePrint (2009/374)