The Lightweight Block Cipher LED as an Inspiration for Cryptanalysis

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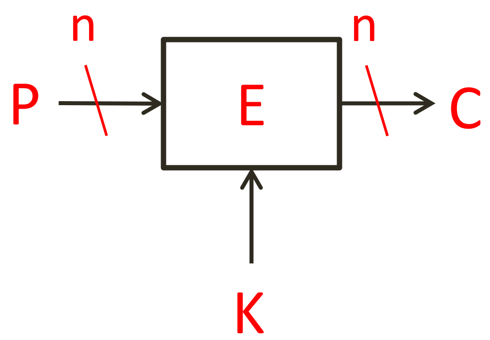
Block Ciphers

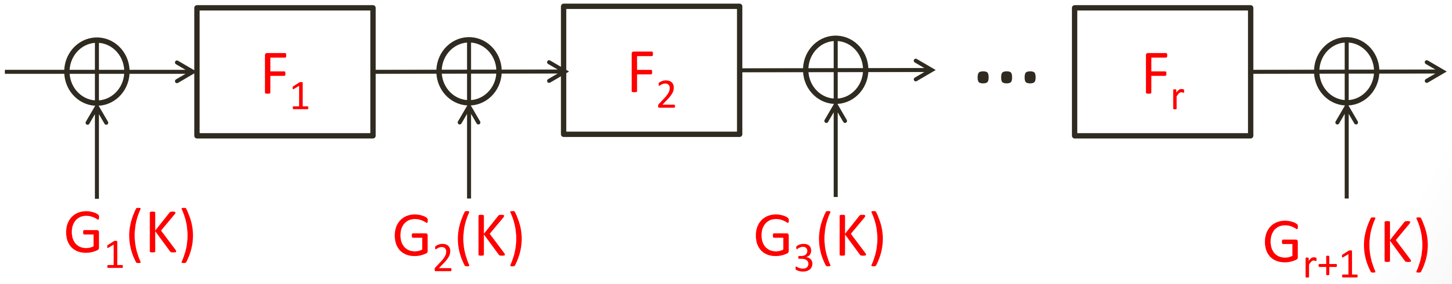


- A collection of permutations over n-bit strings indexed by a secret key K

Block Ciphers

- Block ciphers are traditionally designed with **complex key schedules**

- Ensures **fast diffusion** of key bits into the state
  - Provides enhanced resistance against cryptanalytic attacks such as **meet-in-the-middle**

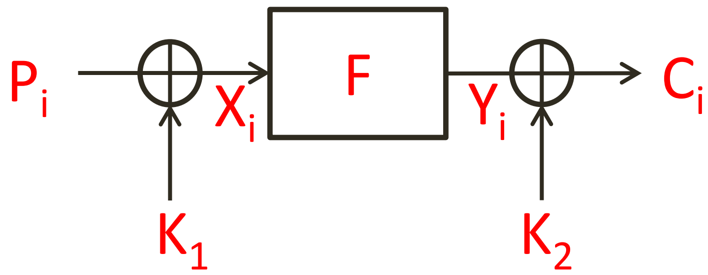


Block Ciphers

- In recent years, with the rise of **lightweight cryptography**, designers are forced to **simplify** the key schedule

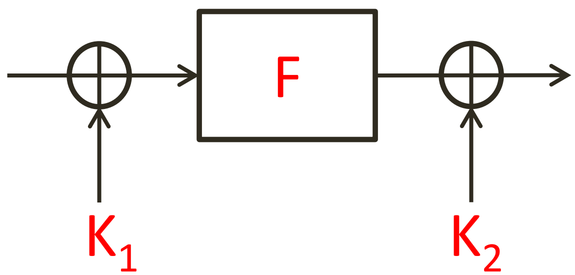
- There is renewed interest in the **Even-Mansour** scheme - A simple construction of a block cipher proposed in 1991

The Even-Mansour Scheme (1991)



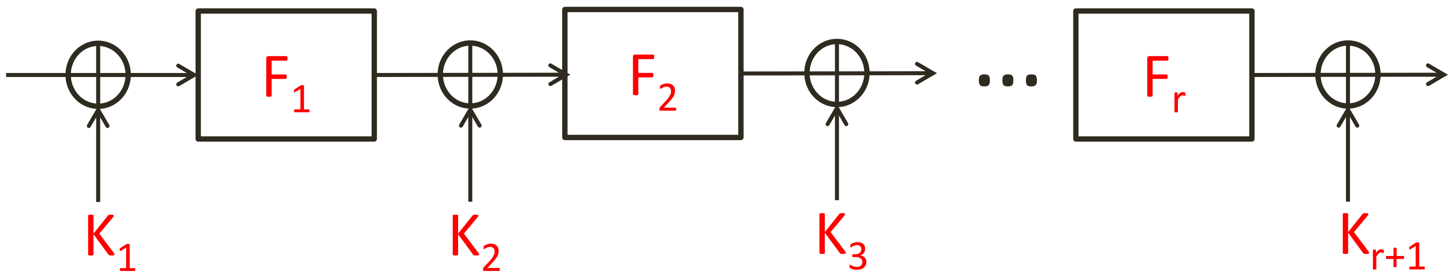
- A simple construction of a block cipher using 2 keys of n bits and a **public** permutation F
- **Information-theoretic** security lower bound:
  - Assume that F is **randomly chosen**
  - Assume that we obtain D plaintext-ciphertext pairs (P_i,C_i)
  - Then, any successful key-recovery attack that evaluates F on T inputs X must satisfy $T \geq 2^n/D$

The SlideX Attack [DKS '12]



- Security: $TD=2^n$ using the **SlideX** attack (DKS, Eurocrypt '12)
- Given $D=2^{n/2}$ the scheme can be broken in $T=2^{n/2}$
  - Considering $D>2^{n/2}$ is less interesting

The Iterated EM Scheme



- EM-based schemes are a **very hot** research area
  - Over 10 papers in major crypto conferences since 2011

- There are many possible **key schedules**

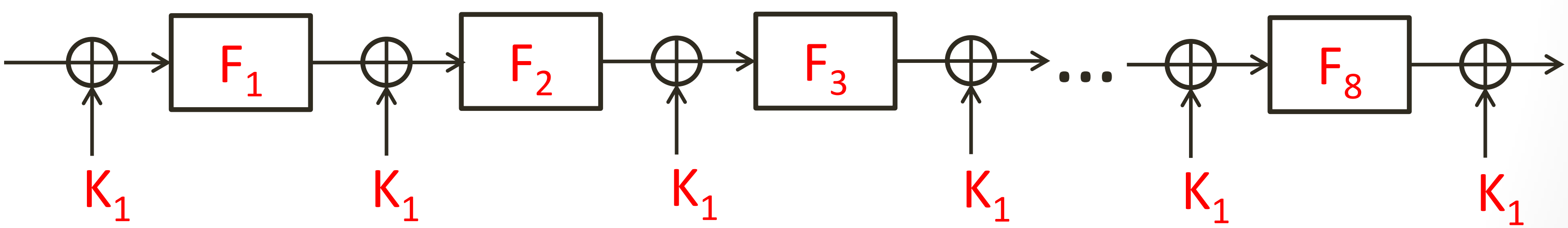
The Lightweight Block Cipher LED

- LED is a 64-bit lightweight block cipher presented at CHES 2011 by Guo et al.

- Two main versions: LED-64 and LED-128

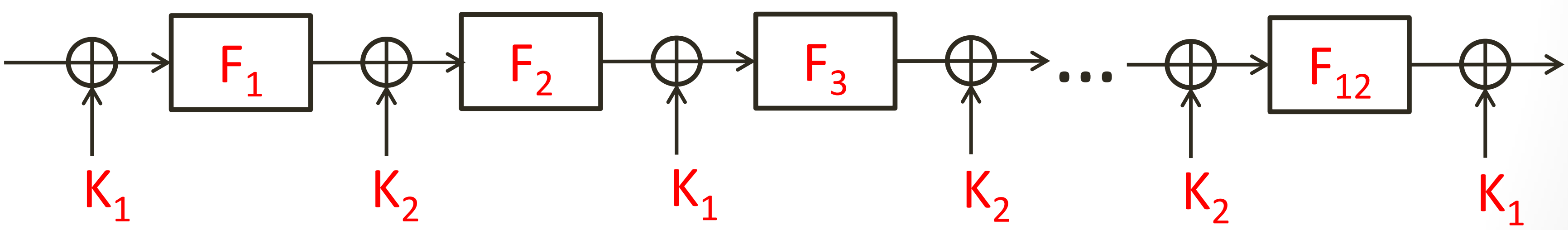
LED-64

- LED-64 is an 8-round EM scheme with 1 key



LED-128

- LED-128 uses 2 alternating keys and has 12 rounds

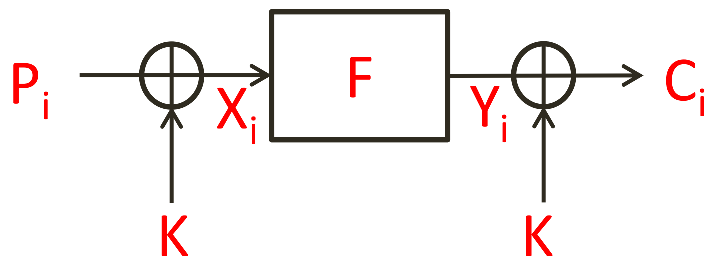


Summary

- Study the security of LED as an **iterated Even-Mansour** scheme

- The **cryptanalytic techniques** do not exploit the properties of the internal permutations of LED
  - Applicable to **any block cipher** that uses a similar key schedule

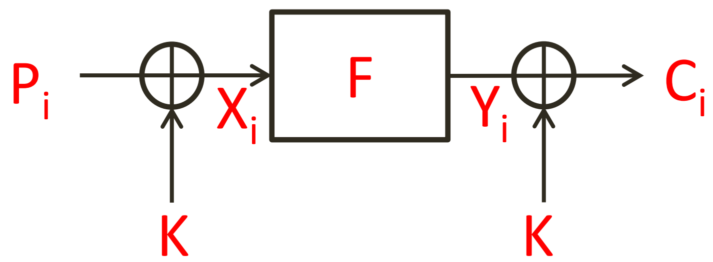
SlideX on EM with 1 Key [DKS '12]



- $P_i+K=X_i$ and $C_i+K=Y_i$ → $P_i+C_i=X_i+Y_i$
- For each (P_i,C_i):
  - Calculate P_i+C_i and store it in a sorted table next to P_i
- For arbitrary values X_j:
  - Calculate $Y_j=F(X_j)$ and search X_j+Y_j in the table
  - For each match, test the suggestion for $K=P_i+X_j$

P_i+C_i	P_i
⋮	⋮

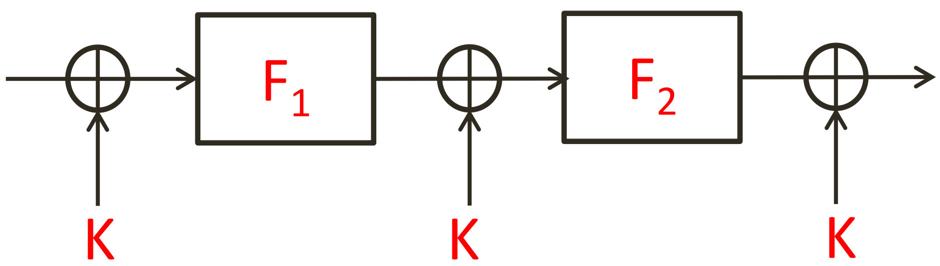
SlideX on EM with 1 Key-Analysis



- The attack succeeds once we have (P_i,X_j) such that $K=P_i+X_j$
- In order to obtain such a pair **w.h.p** we need a total of about 2^n pairs (P_i,X_j), i.e. $TD=2^n$
- For an arbitrary X_j we expect less than one match in the table
  - The time complexity of the attack is $\max(T,D)=T$ (assuming $D \leq 2^{n/2}$)
- The memory complexity M of the attack is equal to D

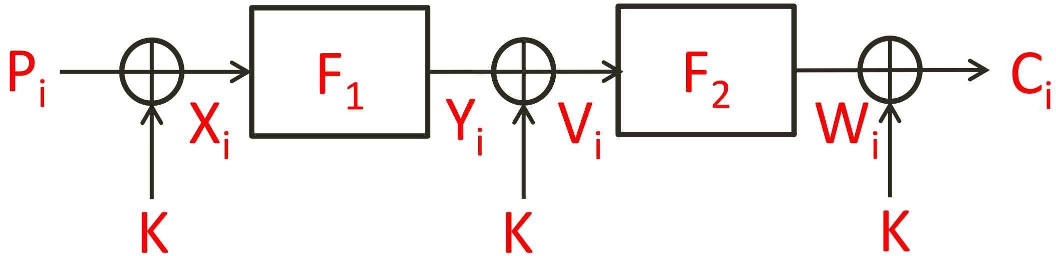
2-Round Iterated EM with 1 Key

- Does not provide n-bit security as shown at FSE 2013 [NWW '13]



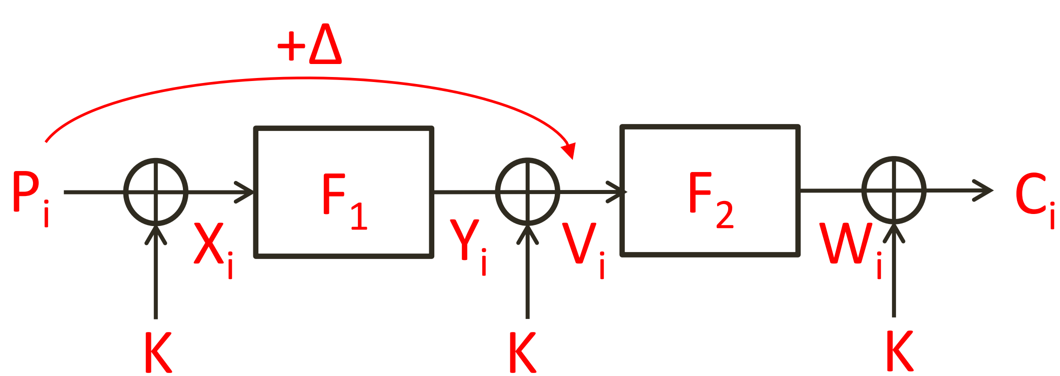
A Variant of the Previous Attack [NWW '13] – Main Idea

- $P_i+V_i=X_i+Y_i$ → $X_1+Y_1= X_2+Y_2=\ldots=X_t+Y_t=\Delta$ then $P_1+V_1= P_2+V_2=\ldots=P_t+V_t=\Delta$
- A t-way collision on the **public** $F'_1(X)=X+F_1(X)$ gives a t-way collision on P_i+V_i with the **same** value Δ
- Given Δ, and a random P_i, then $V_i=P_i+\Delta$ with probability $t/2^n>1/2^n$



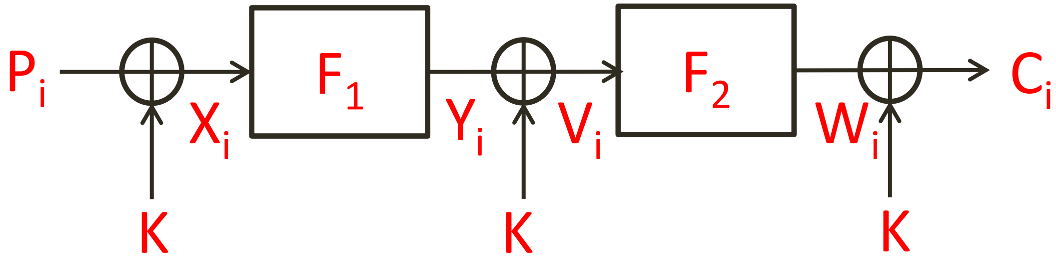
A Variant of the Previous Attack [NWW '13]

- **Preprocessing**: Evaluate F_1 on arbitrary inputs X, find a t-way collision on $F'_1(X)=X+F_1(X)$ and denote the colliding value by Δ
- **Online**: For each (P_i, C_i):
  - Assume that $V_i=P_i+\Delta$ and compute $W_i=F_2(V_i)$
  - Compute a suggestion for $K=W_i+C_i$ and test it



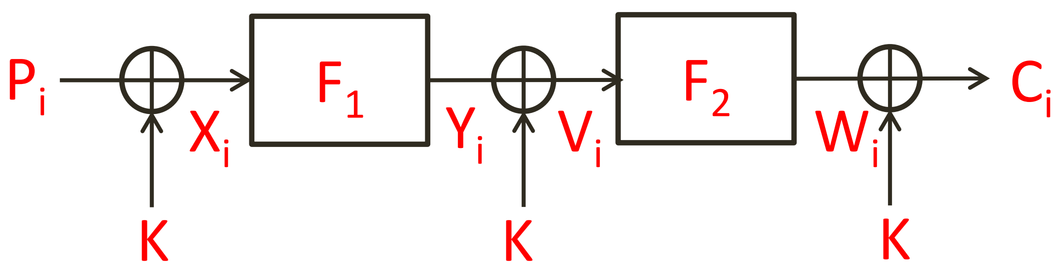
A Variant of the Previous Attack [NWW '13] - Analysis

- The data complexity is $D=2^n/t$
  - in order to find a P_i such that $V_i=P_i+\Delta$ and recover K
- The **online** time complexity is also $2^n/t$
- What is the complexity of the preprocessing?



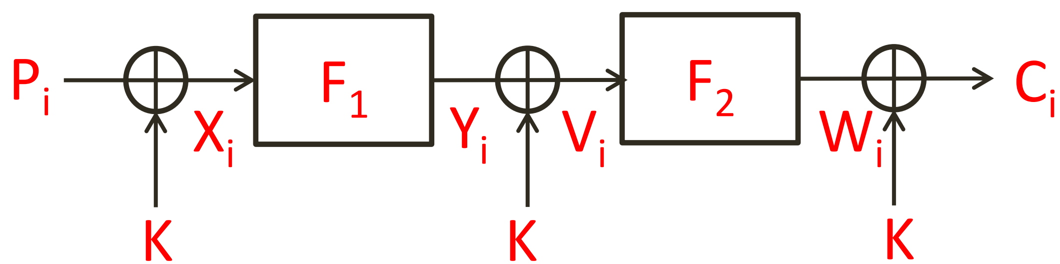
A Variant of the Previous Attack [NWW '13] - Analysis

- If we evaluate F'_1 on **all** 2^n inputs, the attack will not be faster than **exhaustive search**
- We evaluate F'_1 on a $\lambda<1$ fraction of the inputs
- The **preprocessing** time complexity is $\lambda 2^n$
  - in which we find a t-way collision



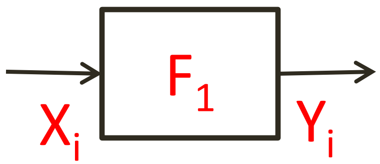
A Variant of the Previous Attack [NWW '13] - Analysis

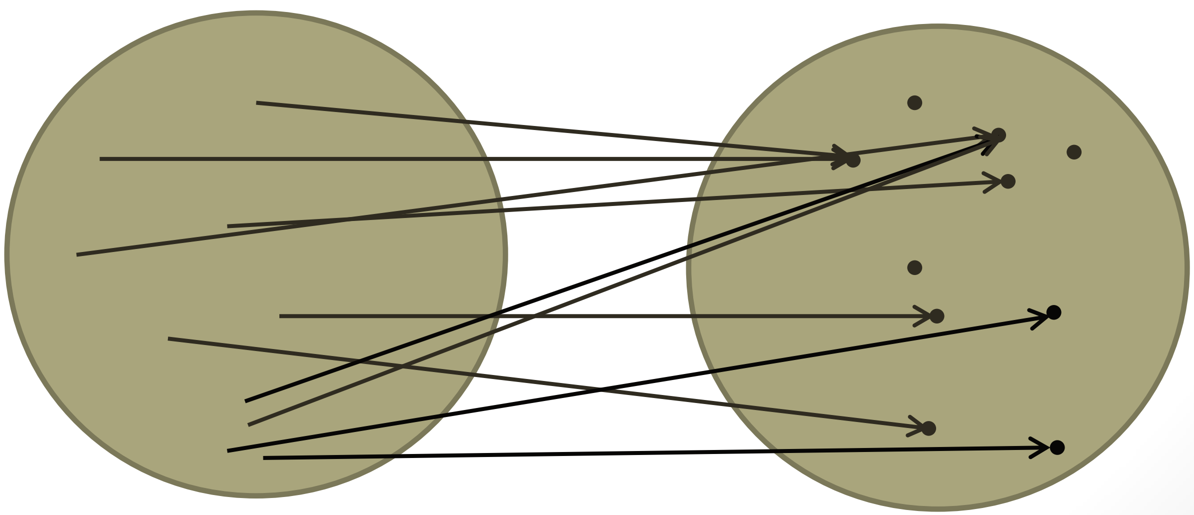
- The **total** time complexity is $\lambda 2^n + 2^n/t$
- To calculate the **optimal** time complexity, we need to understand the **tradeoff** between λ and t
- What is the largest t-way collision we expect when evaluating a λ fraction of inputs for F'_1?



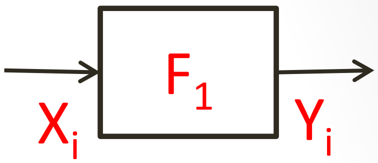
A Variant of the Previous Attack [NWW '13] - Analysis

- $F'_1(X)=X+F_1(X)$ is a function from n bits to n bits
- A function can be described using a **bipartite graph**, mapping inputs to outputs

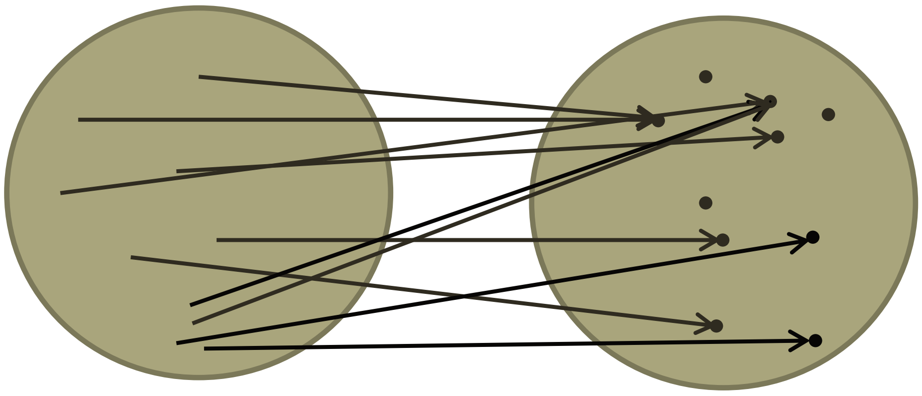




A Variant of the Previous Attack [NWW '13] - Analysis

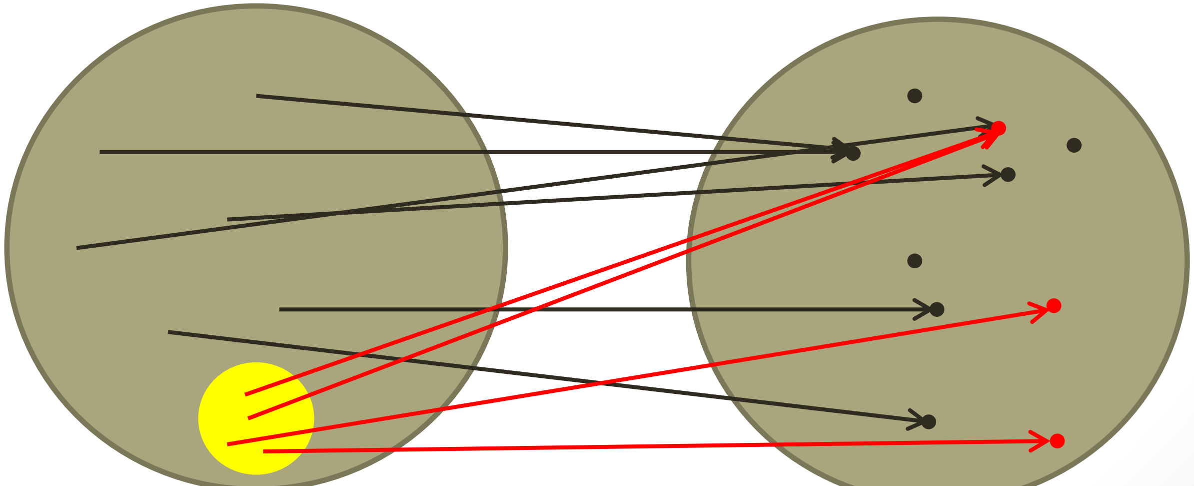


- Assuming F_1 is a **random permutation**, $F'_1(X)=X+F_1(X)$ is (very close to) a **random function** mapping n bits to n bits
- The in-degree of a vertex in the range of F'_1 is distributed according to the **Poisson distribution**
  - The expectation equal to the **average in-degree** λ



A Variant of the Previous Attack [NWW '13] - Analysis

- We expect $(2^n \lambda^t e^{-\lambda})/t!$ vertices with an in-degree of t for F'_1



A Variant of the Previous Attack [NWW '13] - Analysis

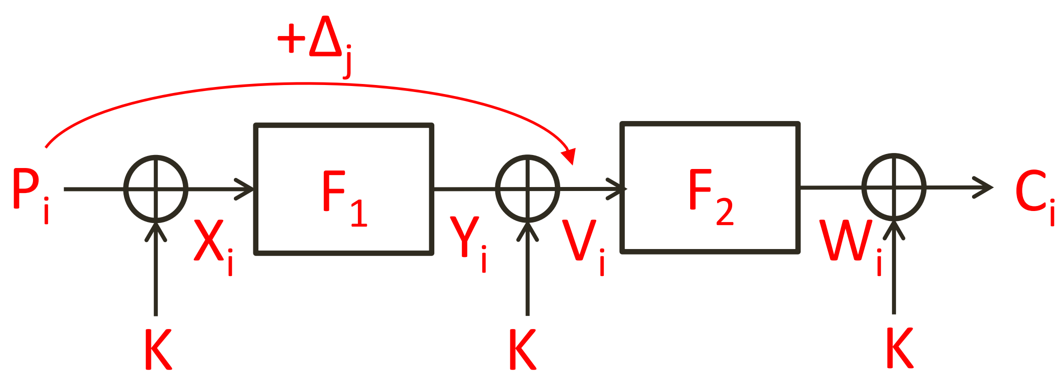
- The **tradeoff** between λ and t is enforced by $(2^n \lambda^t e^{-\lambda})/t! \geq 1$
- Taking $\lambda \approx 1/n$ gives $t \approx 1/\lambda \approx n$ and **minimizes** $T \approx 2^n/n$
  - This is faster than **exhaustive search** by a factor of about n, which grows to **infinity** with n
- For $n=64 \rightarrow T \approx 2^{64}/64 \approx 2^{60}$ and also $D \approx 2^{60}$, $M \approx 2^{60}$

A First Optimization: Reducing the Data Complexity – Main Idea

- Once we take λ and t for which $(2^n\lambda^t e^{-\lambda})/t! \geq 1$, and **slightly** reduce t, the number of t-way collisions grows **rapidly**
- For $n=64$ and 2^{60} inputs we expect:
  - **4** 10-way collisions
  - **95** 9-way collisions
  - Over **100,000** 8-way collisions

A First Optimization: Reducing the Data Complexity

- **Preprocessing**: Evaluate F_1 on arbitrary inputs X, find $t_1, t_{2\dots}, t_l$-way collisions on $F'_1(X)=X+F_1(X)$ and denote the colliding values by $\Delta_1, \Delta_{2\dots}, \Delta_l$
- **Online**: For each (P_i, C_i):
  - For each Δ_j:
  - Assume that $V_i=P_i+\Delta_j$ and compute $W_i=F_2(V_i)$
  - Compute a suggestion for $K=W_i+C_i$ and test it

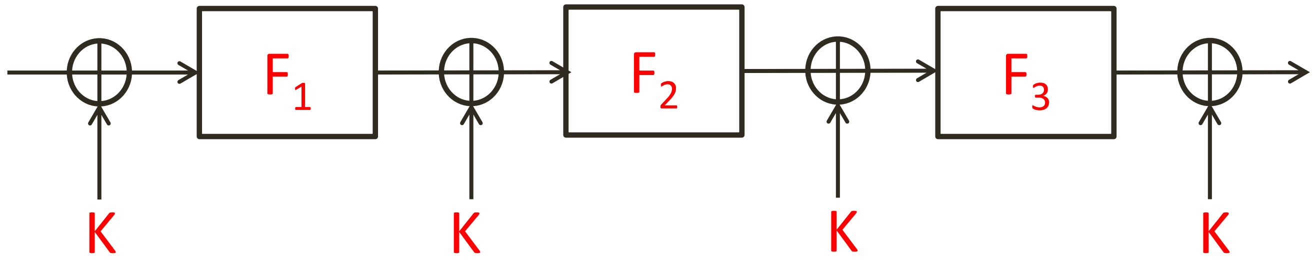


A First Optimization: Reducing the Data Complexity – Analysis

- How much data do we need?
- Let $\text{ţ}=(t_1+t_2\ldots+t_{\text{ļ}})/\text{ļ}$
- The collisions on $\Delta_1,\Delta_{2\ldots},\Delta_{\text{ļ}}$ "cover" ţļ values of X_i for which we need to find a matching P_i
- By the birthday paradox we need **only** $D=2^n/\text{ţļ}\approx 2^n/\max(t_i)\text{ļ}$
- For $n=64$ we **greatly reduce** the data complexity from 2^{60} to 2^{45} by taking $\text{ţ}=8$ rather than $\max(t_i)=10$
  - The time and memory complexities slightly increase but remain about 2^{60}

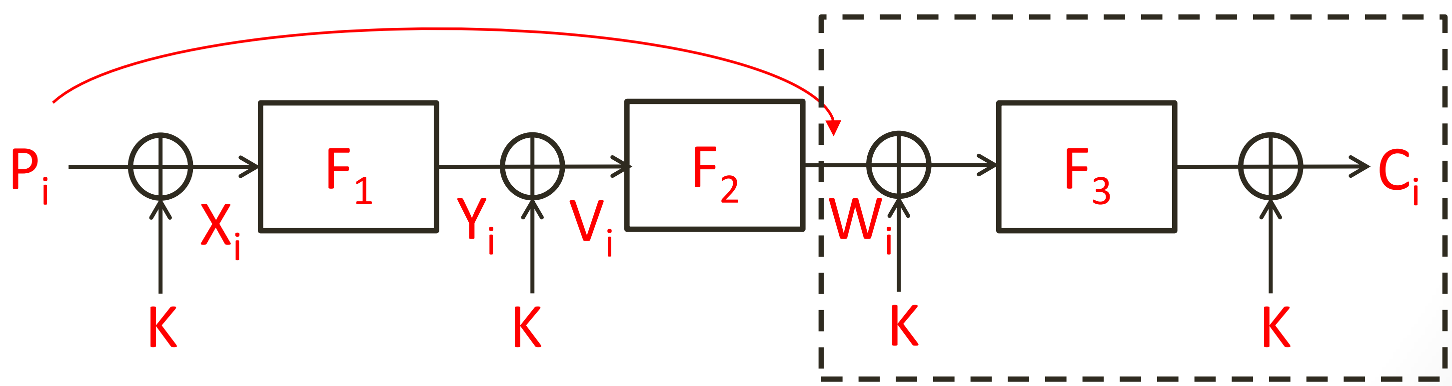
3-Round Iterated EM with 1 Key

- The attack on 2-round EM was already somewhat marginal
- 3-round EM **does not** provide n-bit security as well!



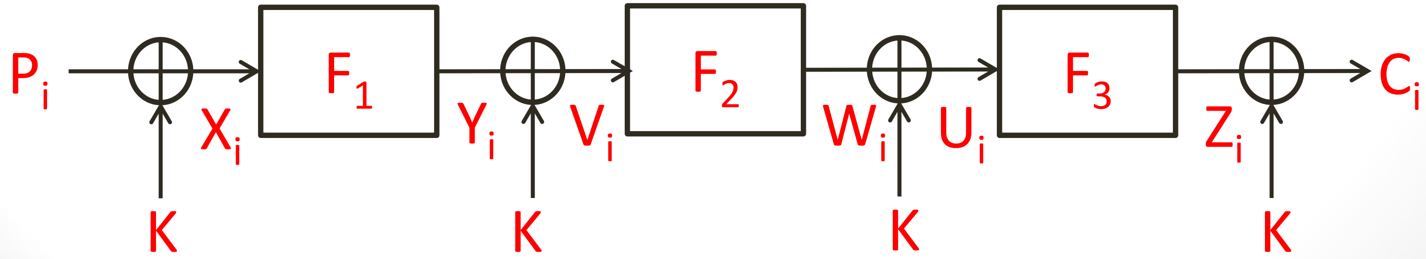
The Main Idea of the Attack

- We know how to predict W_i with a higher probability than a random guess
- Given W_i and C_i we remain with a 1-round EM with 1 key and can apply the **SlideX** attack



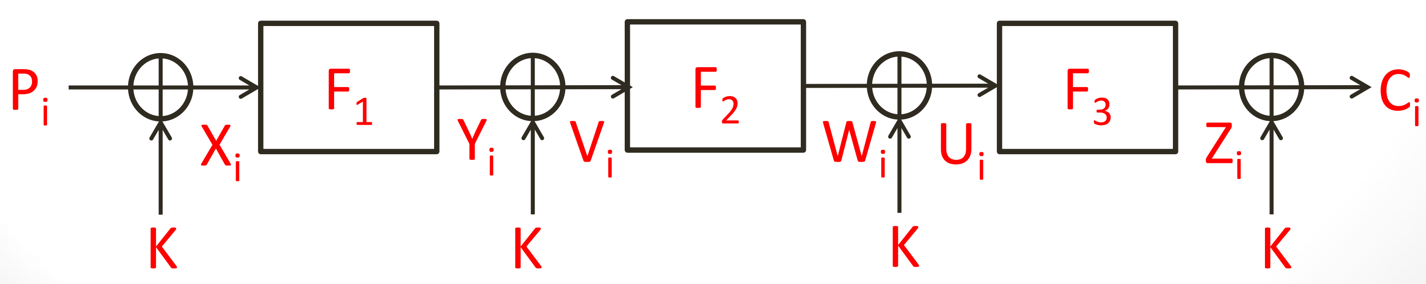
The Basic 3-Round Attack

- **Preprocessing**: Find a t-way collision on $F'_1(X)$ and denote the colliding value by Δ
- Evaluate F_3 on inputs U and store $U+Z$ in a sorted table
- **Online**: For each (P_i, C_i):
  - Assume that $V_i=P_i+\Delta$ and compute $W_i=F_2(V_i)$
  - Search W_i+C_i in the table
  - For each match obtain Z_i and test $K=Z_i+C_i$



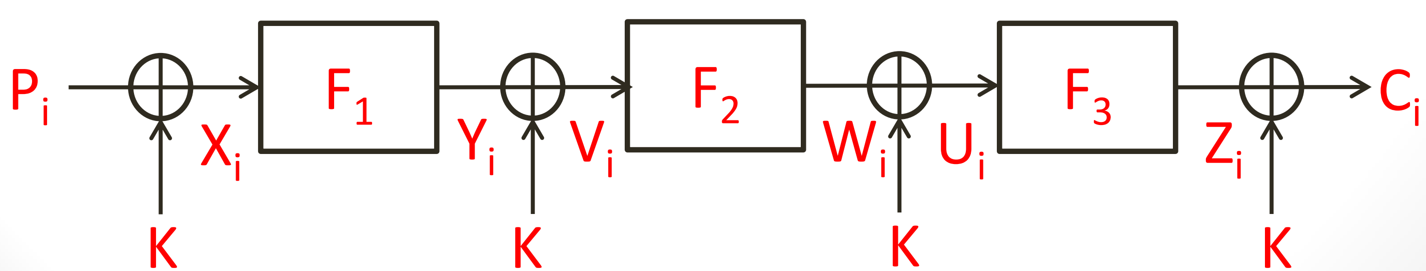
The Basic 3-Round Attack - Analysis

- For each (P_i,C_i) the probability that $W_i=F_2(P_i+\Delta)$ is $t/2^n$
- We expect to correctly calculate W_i for $Dt/2^n$ plaintexts
- The table of F_3 needs to have $2^{2n}/Dt$ entries
- The **time complexity** of the attack is $\lambda 2^n+(1/D)(2^{2n}/t)+D$
- The **optimal** attack is obtained for $D\approx 2^n/\sqrt{t}$, which gives $T\approx 2^n/\sqrt{n}$
  - Faster than **exhaustive search** only by a factor of $\sqrt{n}$



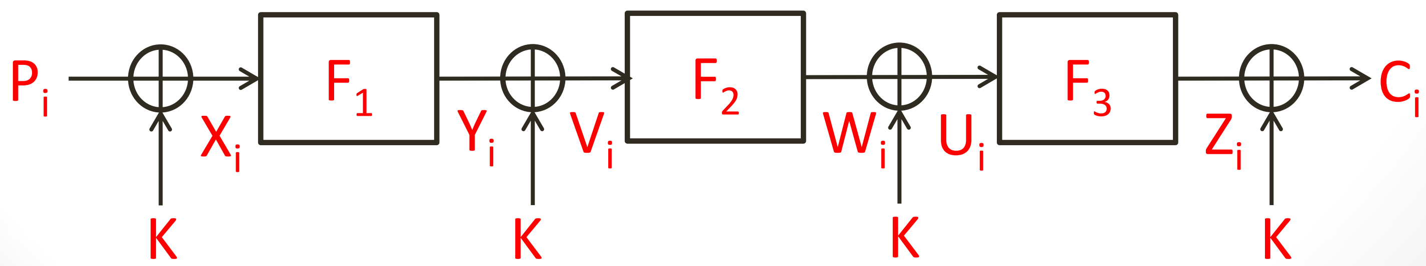
Optimizing the 3-Round Attack

- Apply the same optimization as in the 2-round attack to reduce the **data complexity**
- Use the **freedom** to choose the inputs on which we evaluate F_1 and F_3 in order to reduce the **time complexity**



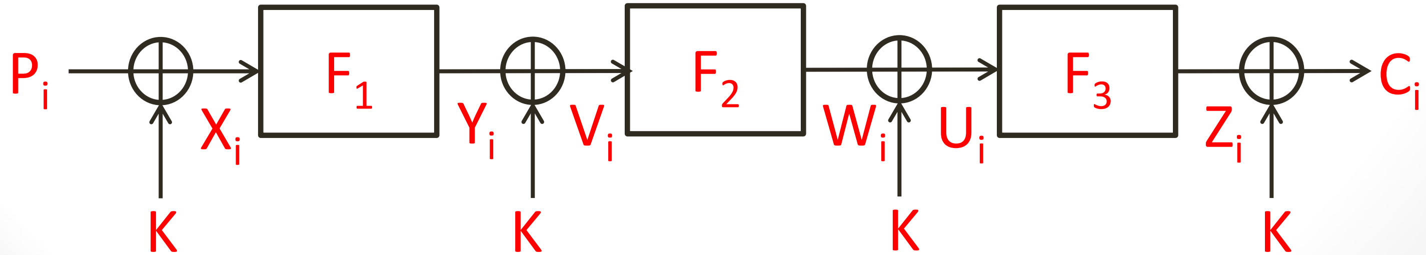
Optimizing the 3-Round Attack

- $P_i + C_i = X_i + Z_i$ and in particular, this holds if we only consider the m **most significant bits**
- Evaluate F_1 on inputs X where the m MSBs are 0
- Evaluate $(F_3)^{-1}$ on inputs Z where the m MSBs are 0
- For (P_i, C_i) if we evaluated F_1 on X_i **and** we evaluated $(F_3)^{-1}$ on Z_i **then** the m MSBs of P_i and C_i must be equal!
- This allows us to **immediately filter** most (P_i, C_i)



The Optimized 3-Round Attack

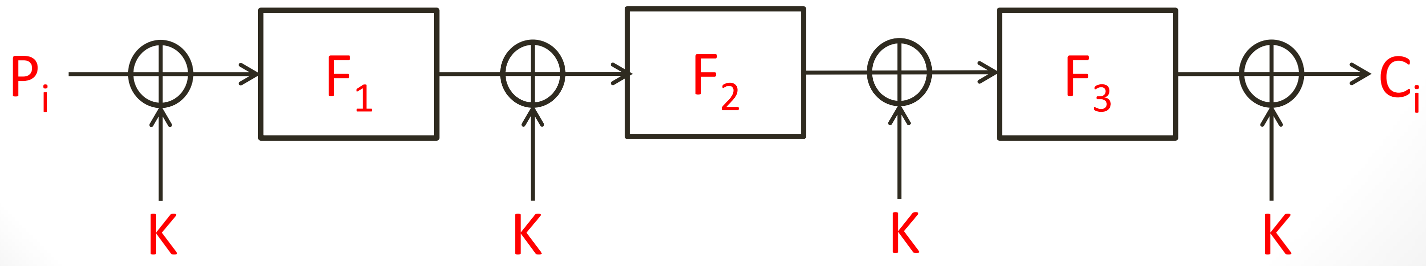
- The optimization gives us $T \approx 2^n/n$
- This is about the **same** time complexity as the 2-round attack!



Application to LED-64

- LED-64 is an 8-round EM scheme with 1 key

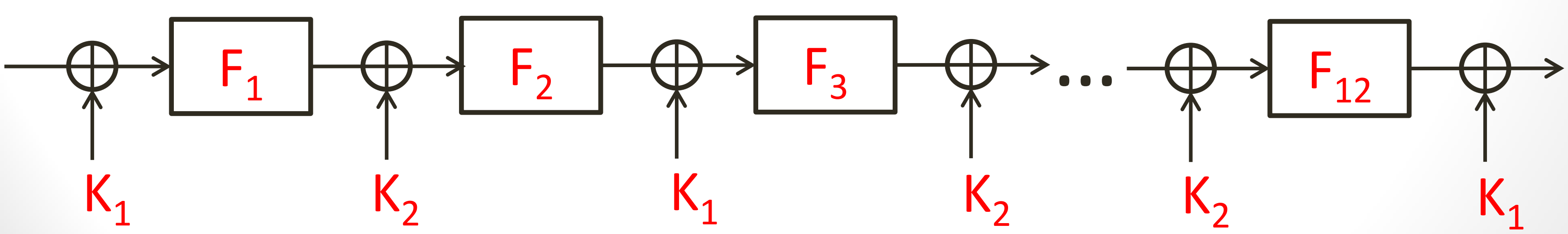
- We can directly apply our attack to 3-round LED-64 with $T \approx 2^{60}$, $M \approx 2^{60}$ and $D = 2^{49}$



Application to LED-128

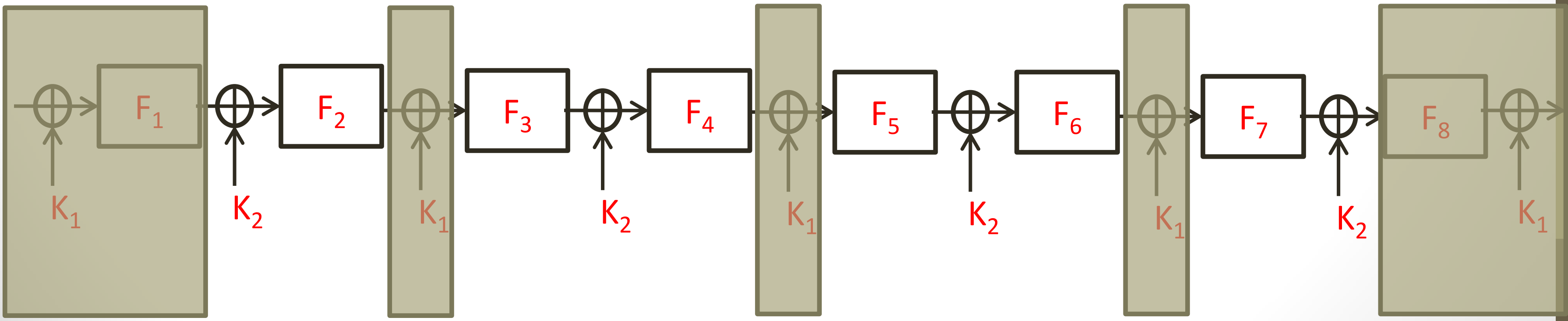
- LED-128 uses 2 alternating keys and has 12 rounds

- We use the new techniques to attack 8 rounds!



Application to LED-128

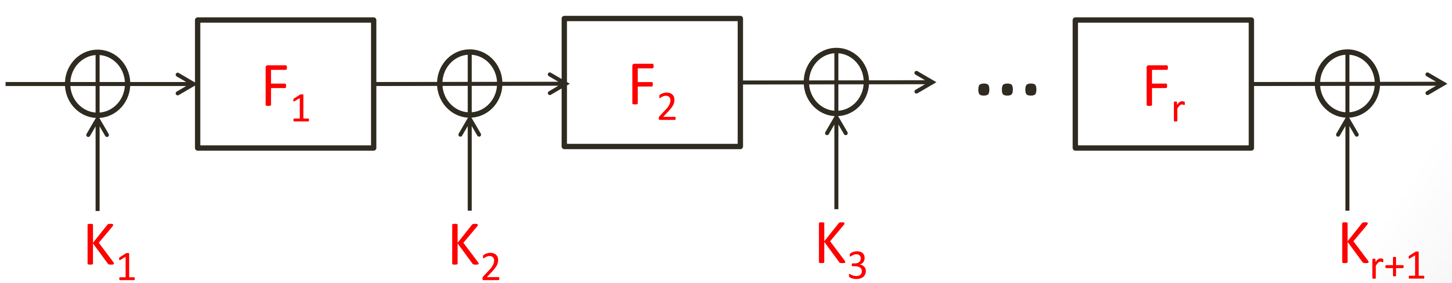
- Guess K_1 in an outer loop
- We remain with a 3-round EM scheme with 1 key
- We obtain $T \approx 2^{124}$, $M \approx 2^{60}$ and $D = 2^{49}$



Involutions

- In practice the permutations F_i can be constructed using a block cipher without the **key schedule**
- Many of these constructions have the property that they are **equal to their inverses**

- A permutation F is called an **involution** if $F=F^{-1}$

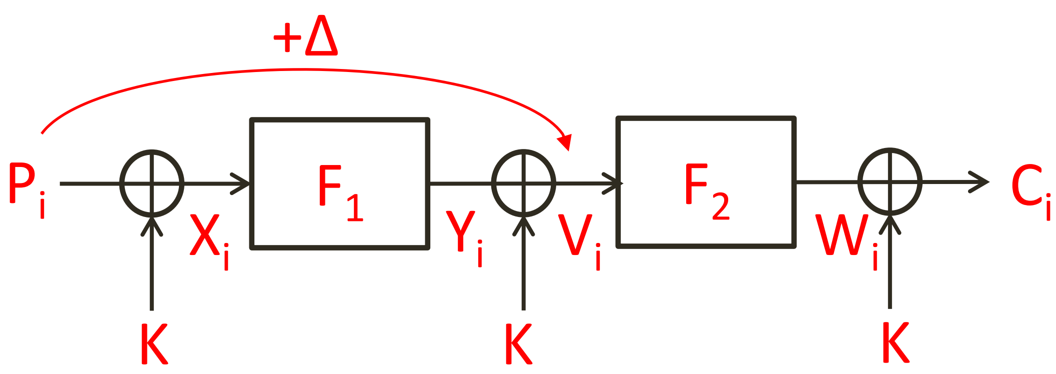


Fixed Points of Involutions

- A **random** involution has an expected number of $2^{n/2}$ **fixed-points**
- $x=F(x)$ → $F'(x)=x+F(x)=0$ → the vertex 0 in the graph of $F'(x)$ has an expected in-degree of $2^{n/2}$
  - This is much **larger** than the $O(n)$ in-degree of a vertex of **maximal** in-degree, when F is a random **permutation**

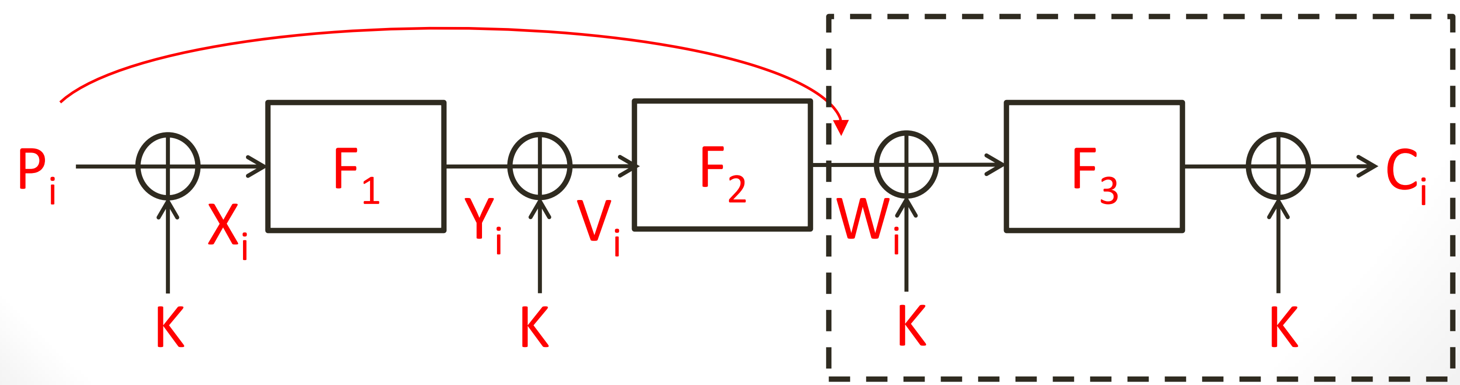
Applications to Iterated EM

- A 2-round iterated EM scheme with 1 key can be attacked in $T \approx 2^n/t$
- When F_1 and F_2 are random permutations $T \approx 2^n/n$
- When F_1 (or F_2) is a **random involution** $T \approx 2^{n/2}$
  - The memory and data complexities are also significantly reduced



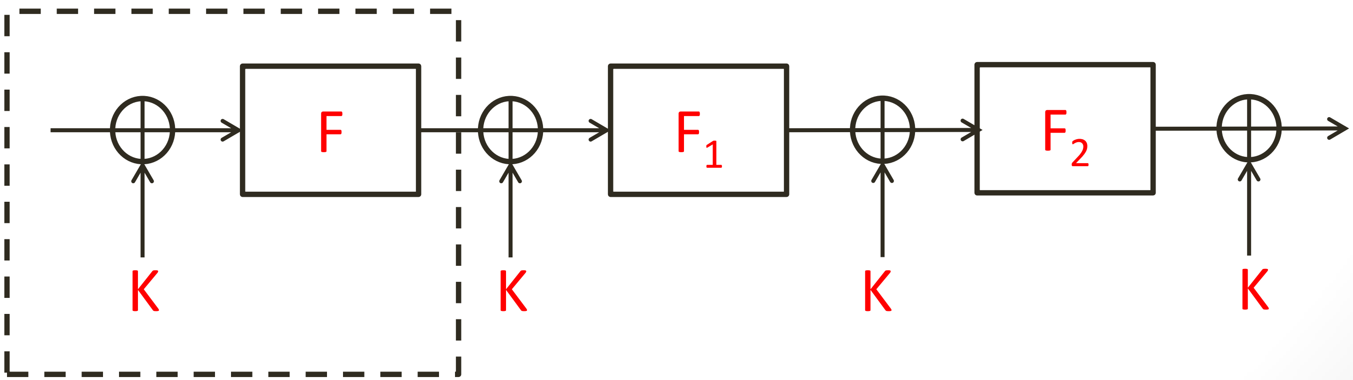
Applications to Iterated EM

- A 3-round iterated EM scheme with 1 key can be attacked in $T \approx 2^n/\sqrt{t}$
- When all permutations are random $T \approx 2^n/\sqrt{n}$
- When F_1 (or F_2 or F_3) is a **random involution** $T \approx 2^{3n/4}$
  - The memory and data complexities are also significantly reduced



A Surprising Application

- A 2-round iterated EM scheme with 1 key with **random permutations** can be attacked in $T \approx 2^n/n$
- Add an **arbitrary involutional round** (unrelated to the original permutations)
- This **significantly reduces** the security to $T \approx 2^{3n/4}$!!
  - Also significantly reduces the data and memory complexities of the attack



Conclusions

- We presented attacks on several schemes based on iterated Even-Mansour
- We attacked 3 out of 8 rounds of LED-64
- We attacked 8 out of 12 rounds of LED-128
- The attacks can be applied to several other block ciphers such as Zorro and AES^2
- The attacks are **unlikely** to be practically significant
- They show that a 1-key EM scheme needs to have **at least** 4 rounds to provide n-bit security

Thank you for your attention!