A Unified Algorithm for Accelerating Edit-Distance Computation via Text-Compression

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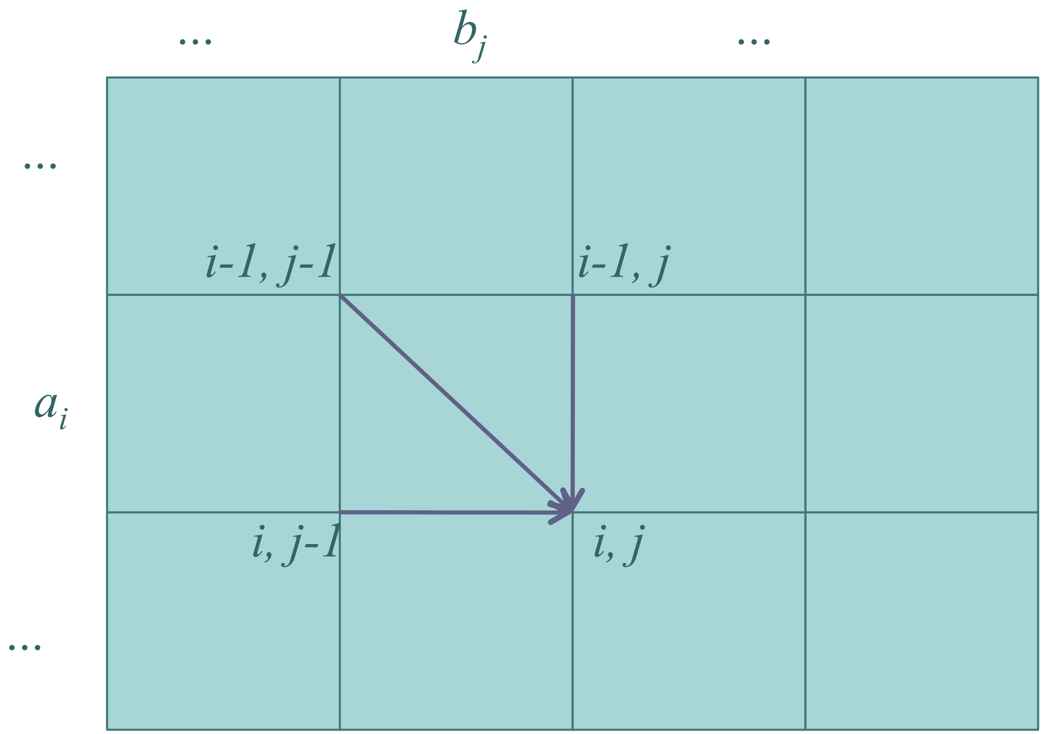
Edit Distance: Quick Review

- The min cost of transforming one string into another via insertion/deletion/replacement.
- One of the fundamental problems in computer science.
- Standard solution: dynamic programming (DP). Time complexity on strings of length N: $O(N^2)$.
- Recent approximation algorithms: Rabani *et al.*

Edit Distance: Quick Review

$$T[i,j] = \min \begin{cases} T[i-1,j] + \text{cost of deleting } a_i \\ T[i,j-1] + \text{cost of inserting } b_j \\ T[i-1,j-1] + \text{cost of replacing } a_i \textit{ with } b_j \end{cases}$$



Acceleration via Compression

- Use compression to accelerate the above DP solution
- Basic idea:
  1. Compress the strings
  2. Compute edit-distance of compressed strings

Acceleration via Compression

N = total length of strings
n = length of compression

- Run-Length encoding
  - Bunke and Csirik ’95
  - Series of results: Apostolico et al. $O(n^2 lgn)$ for LCS. Arbel et al. $O(nN)$ for edit-distance.
- LZW-LZ78
  - Crochemore et al ‘03
    - $O(nN)$
    - Constant size alphabets: $O(N^2/logN)$
- Masek, Paterson ’80
  - Exploit repetitions + “Four-Russians technique” $O(N^2/log^2N)$ for any strings, rational scoring function
  - Bille, Farach-Colton ‘05 extend to general alphabets

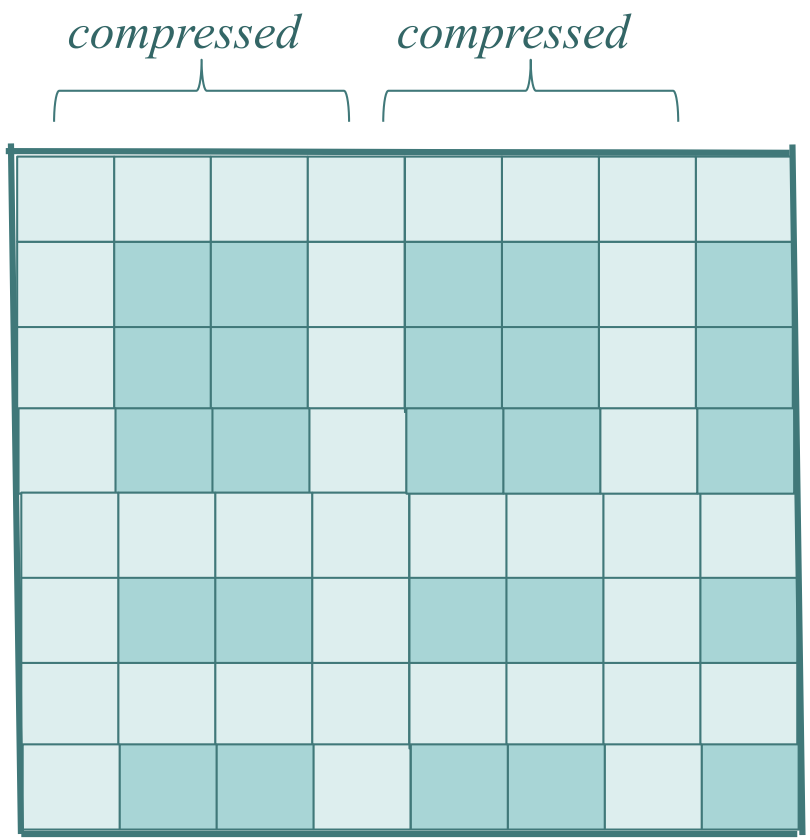
A Unified Acceleration

- Find a general compression-based edit distance acceleration for *any* compression scheme…
- Can handle two strings that compress well on different schemes
- Towards breaking the quadratic barrier of edit-distance computation

A Unified Acceleration

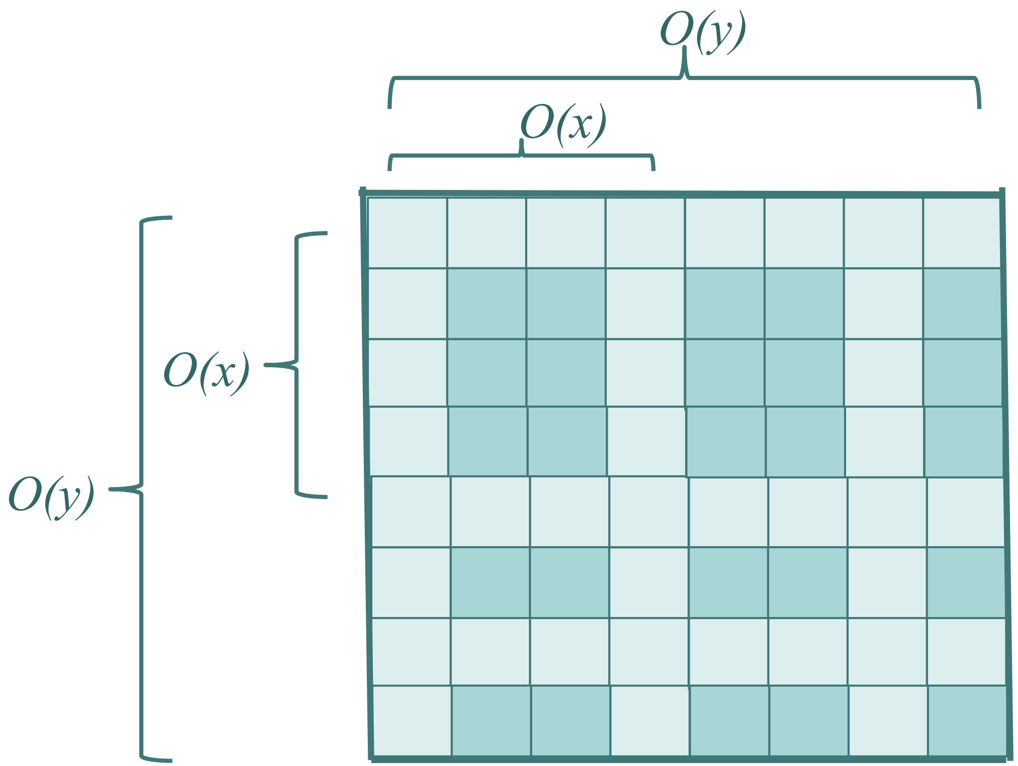
- Basic idea of the Crochemore *et al.* algorithm

1. Divide DP-grid into blocks
2. Build a repository of DIST tables for all blocks
3. Compute edit distance by computing boundaries of each block
   - propagate DP-values using SMAWK



A Unified Acceleration

- Definition: *xy-partition* of G:
  - Partitioning of G into blocks:
    - Boundary size of blocks: $O(x)$
    - $O(y)$ blocks in each row and each column

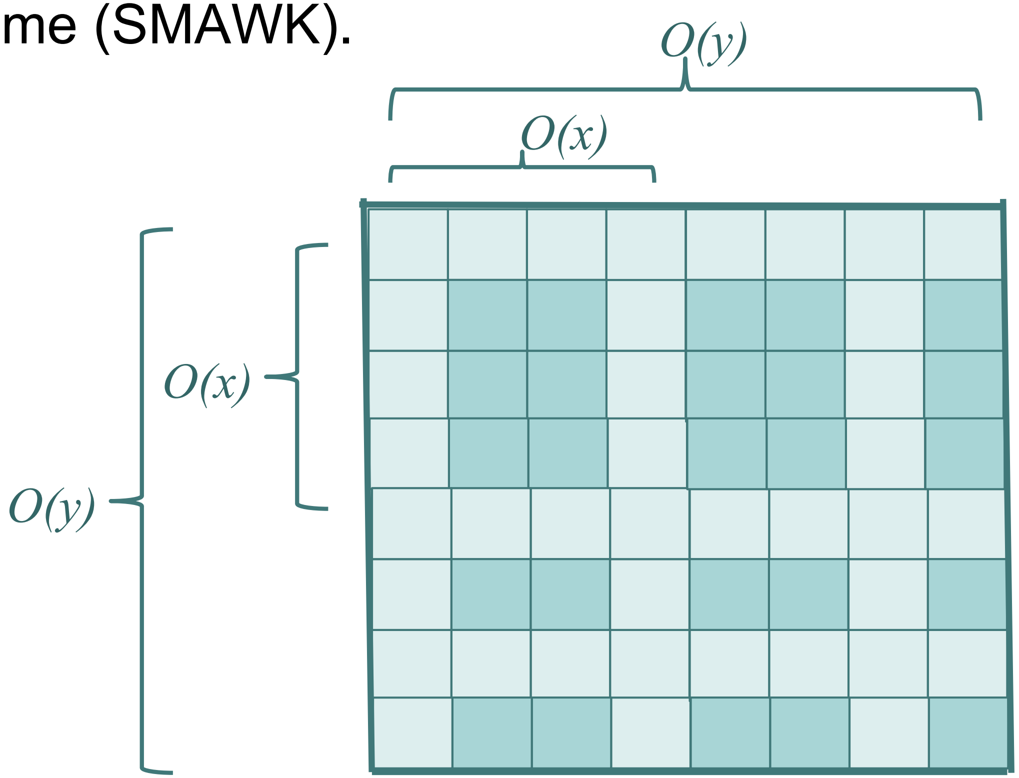


A Unified Acceleration

- Running-time:
  - Constructing the repository:
    - #DIST × $O(x^2 lgx)$ time (Apostolico et al. '90)
  - Propagating the DP-values:
    - $O(Ny)$ time (SMAWK).

hopefully many repetitions

N = total length of strings
n = length of compression



A Unified Accelerator

- Find a good xy-partition for any pair of compressible strings.
- How can we achieve this?

Using Straight-Line Programs

Straight-line Programs (SLP)

- Context-free grammar
- Every grammar generates exactly one string
- Allow 2 types of productions:
  - $X_i \rightarrow a$ (a is a unique terminal)
  - $X_i \rightarrow X_p X_q \ (i > p,q)$

Straight-line Programs (SLP)

Example: S=abaababaabaab

Use Fibonacci SLP:

$X_1 \rightarrow a$

$X_2 \rightarrow b$

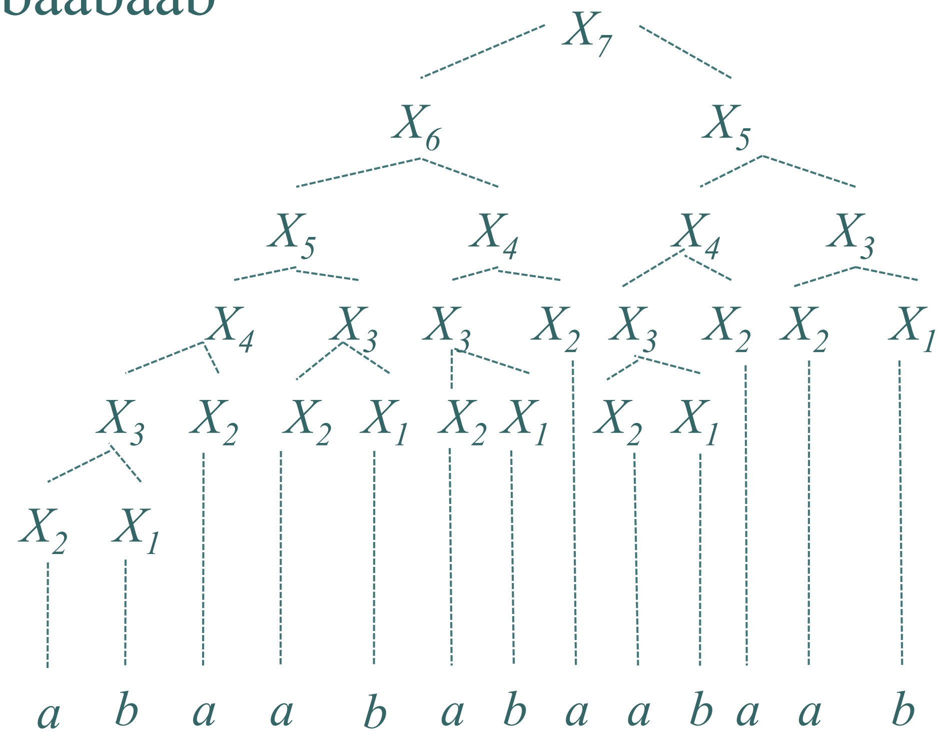
$X_3 \rightarrow X_2X_1$

$X_4 \rightarrow X_3X_2$

$X_5 \rightarrow X_4X_3$

$X_6 \rightarrow X_5X_4$

$X_7 \rightarrow X_6X_5$



Straight-line Programs (SLP)

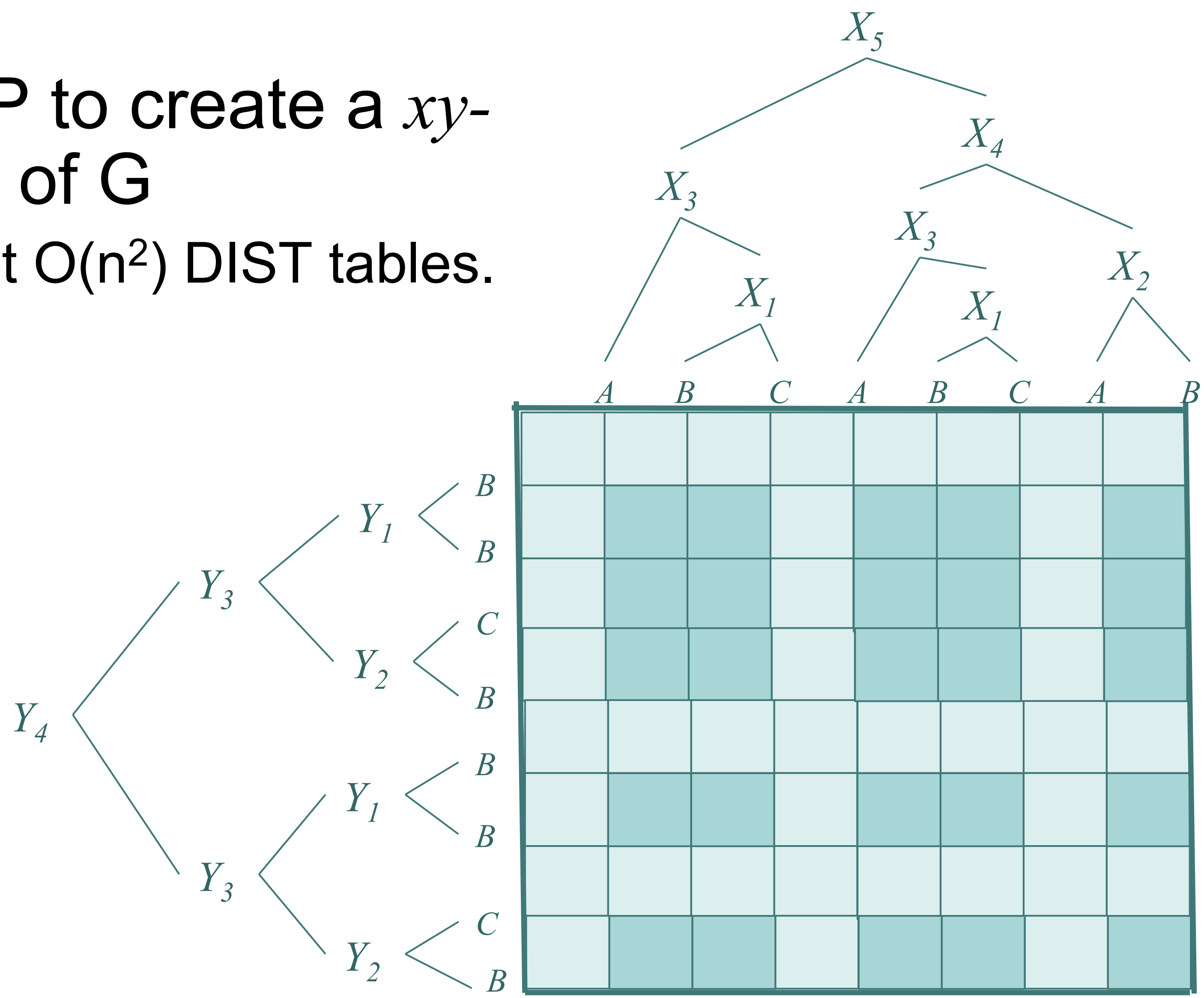
- Why SLP?
  - Result of most compression schemes can be transformed into SLP (Rytter '03)
    - LZ, RLE, Byte-Pair, Dictionary methods…
    - Compressed approximation:
      - String length: N
      - Encoding produces n blocks
      - Get SLP of size $m=O(n\log N)$ in $O(m)$ time
      - m within $\log N$ factor from minimal SLP

Straight-line Programs (SLP)

- Rytter, Lifshits - used SLP for accelerating pattern matching via compression
- Lifshits –
  - various hardness results for SLP e.g.: edit-distance, Hamming distance
  - $O(n^3)$ for determining equality of SLPs
- Tiskin –
  - $O(nN^{1.5})$ algorithm for computing longest common subsequence between two SLPs
  - Can be extended at constant factor to compute edit distance between SLPs

Constructing the xy-partition

- Use SLP to create a *xy-partition* of G
  - At most O(n²) DIST tables.

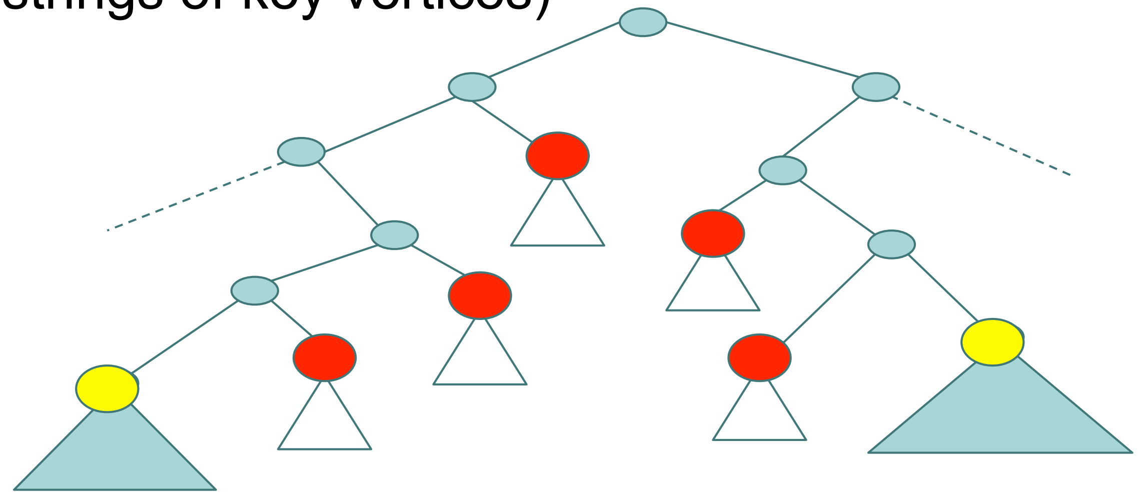


Constructing the xy-partition

- For any x, we can construct an *xy-partition* with $y=O(nN/x)$ in $O(N)$ time.
  - We will choose x later.
  - Use SLP parse tree.

Constructing the xy-partition

- Choose nN/x “key vertices” in tree s.t. each vertex is variable generating substring of length $O(x)$
    1. Find $O(N/x)$ variables in A' generating disjoint substrings of length between x and $2x$
    2. Substrings in A not yet covered can be generated using $O(n)$ additional variables for each 2 found in step 1
    3. Total – $O(nN/x)$ vertices (A is the concatenation of all generated substrings of key vertices)



Putting it all together

- Using SLP to compute edit-distance
    1. *Create xy-partition of G according to SLP*
    2. *Build a repository of DIST tables of blocks in xy-partition*
    3. *Compute edit distance by computing boundaries of each block (propagate DP values using SMAWK)*

- Total running time $O(n^2x^2 lgx+Ny)$

constructing repository of all DIST tables

propagating DP values

Putting it all together

- Total running time $O(n^2x^2 lgx+Ny)$

constructing repository of all DIST tables

propagating DP values

- For all x we can build *xy-partition* with $y=O(nN/x)$.
- Choose x so as to balance both terms above.
- Total: $O(n^{1.34}N^{1.34})$ *time.*

Extensions

1. $O(n^{1.4}N^{1.2})$ time for rational Scoring:
   - use recursive construction of DIST tables, compute repository in $O(n^2x^{1.5})$
   - Based on:
     - $x*x$ DIST table stored succinctly in $O(x)$ space (Schmidt)
     - This allows to merge 2 DIST tables in $O(x^{1.5})$ time (Tiskin)
2. Arbitrary scoring and "Four Russians":
   - $\Omega(\lg N)$ speedup for any string (not necessarily compressible)
   - Short enough substrings must appear many times (Masek and Paterson)
   - With SLP we expand this idea to arbitrary scoring functions

Thank You!!!