Quad Trees
Gaussian pyramids
Laplacian Pyramids
Wavelet Pyramids
Applications

Quad tree image representation = a tree representation which represents recursive subdivisions of an image.

Example:
Quad tree representation of an image

Image
Quad Tree representation

1 2
3 4
Quad Tree Applications:

- Compression
- Segmentation (Split & Merge)
- Smoothing
- Binary Image Operations ("And", "Or", "Not")

Quad Tree Representation - Example

Quad Tree Representation - Example
Quad Tree Representation

Binary Operations Using Quad Trees

Original

Threshold = 0.20

Threshold = 0.40

Threshold = 0.55

And

And

Not

Image Pyramids

Image features at different scales require filters at different scales.

Edges (derivatives):

Objects (correlation):

\[ f(x) \]

\[ f'(x) \]
Image Pyramids

Image Pyramid = Hierarchical representation of an image
Image Pyramid = A collection of images at different resolutions.

Low Resolution

High Resolution

Details in image - low+high frequencies

No details in image - low frequencies

Image Pyramid

Low resolution

High resolution

Image Pyramid Frequency Domain

Low resolution

High resolution
Image Blurring = low pass filtering

Image Pyramid

Low resolution

High resolution
Gaussian Pyramid

Burt & Adelson (1981)

Normalized: \( \Sigma w_i = 1 \)

Symmetry: \( w_i = w_{-i} \)

Unimodal: \( w_i > w_j \) for \( 0 < i < j \)

Equal Contribution: for all \( j \) \( \Sigma w_{j+2i} = \) constant

For \( a = 0.4 \) most similar to a Gaussian filter

\[ g = [0.05 \ 0.25 \ 0.4 \ 0.25 \ 0.05] \]

\[ \text{low_pass_filter} = g \ast g' = \]

\[
\begin{array}{cccccc}
0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \\
0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\
0.0200 & 0.1000 & 0.1600 & 0.1000 & 0.0200 \\
0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\
0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \\
\end{array}
\]
Hierarchical Pattern Matching

Pattern matching using Pyramids - Example

<table>
<thead>
<tr>
<th>image</th>
<th>pattern</th>
<th>correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gaussian Pyramid - Computational Aspects

Memory:
$2^N \times 2^N \left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right) = 2^N \times 2^N \times \frac{4}{3}$

Computation:
Level $i$ can be computed with a single convolution with filter $h_i = g \ast g \ast g \ast \ldots$ $i$ times

Example:
$h_g = $
Laplacian Pyramid

Compression:
- Compression rates are higher for predictable values, e.g., values around 0.

\[ G_0, G_1, \ldots \] = the levels of a Gaussian Pyramid.

Predict level \( G_l \) from level \( G_{l+1} \) by Expanding \( G_{l+1} \) to obtain \( G'_l \).

\[ G_l = G'_l \]

Reduce \( G'_l \) to obtain \( G_l \).

Denote by \( L_l \), the error in prediction:

\[ L_l = G_l - G'_l \]

\[ L_0, L_1, \ldots \] = the levels of a Laplacian Pyramid.

Laplacian Pyramid - Example
Laplacian Pyramid - No scaling

Gaussian Pyramid

Original Image

Reconstruction of the original image from the Laplacian Pyramid
Laplacian Pyramid - Computational Aspects

Memory:

\[ 2^n \times 2^n \times (1 + 1/4 + 1/16 + ...) = 2^n \times 2^n \times 4/3 \]

However coefficients are highly compressible.

Computation:

\[ L_i \text{ can be computed from } G_0 \text{ with a single convolution with filter: } k_i = h_{i-1} - h_i \]

Multiresolution Spline

When splining two images, transition from one image to the other should behave:

- High Frequencies
- Middle Frequencies
- Low Frequencies

Wavelet Decomposition

Fourier Space
<table>
<thead>
<tr>
<th>Transform</th>
<th>Basis</th>
<th>Frequency</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier</td>
<td>Sines+Cosines</td>
<td>Not localized in space Not localized in Frequency</td>
<td></td>
</tr>
<tr>
<td>Gaussian Pyramid</td>
<td>Gaussian Filters</td>
<td>Localized in space Not localized in Frequency</td>
<td></td>
</tr>
<tr>
<td>Laplacian Pyramid</td>
<td>Laplacian Filters</td>
<td>Localized in space Not localized in Frequency</td>
<td></td>
</tr>
<tr>
<td>Wavelet Pyramid</td>
<td>Wavelet Filters</td>
<td>Localized in space Localized in Frequency</td>
<td></td>
</tr>
</tbody>
</table>

Image pyramid levels = Filter then sample.

Filters:
- Gaussian Pyramid
- Laplacian Pyramid
- Wavelet Pyramid