Image Processing - Lesson 3

Image Enhancement

- Image Enhancement - Spatial Domain
  - Smoothing filter
  - Median filter
- Convolution
  - 1D Discrete
  - 1D Continuous
  - 2D Discrete
  - 2D Continuous
- Sharpening filter

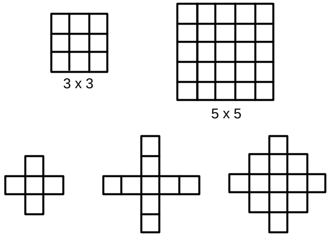
Salt & Pepper Noise

Neighborhood Averaging

S = neighborhood of pixel (x,y)
M = number of pixels in neighborhood S

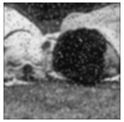
$$\boxed{g(x,y) = (1/M) \sum_{(n,m) \in S} f(n,m)}$$

Neighborhoods:



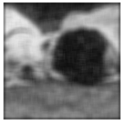
Neighborhood Averaging - Example

Salt & Pepper Noise



3 X 3 Average

5 X 5 Average



7 X 7 Average

Median

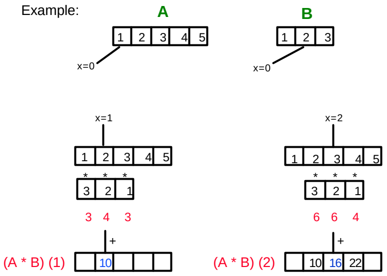
Convolution

A, B = images
B is typically smaller than A and is called the **mask**.

1 dimensional:

$$(A * B)(x) = \sum_{i} A(i)B(x-i)$$

Example:



What happens near the edges?

Convolution with

1	$\overset{\circ}{2}$	3

- Option 1: Zero padding

0	0	0	1	2	3	4	5	0	0	0
			4	10	16	22	22			

- Option 2: Wrap around

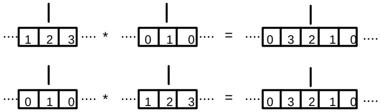
3	4	5	1	2	3	4	5	1	2	3	4	5
			19	10	16	22	23					

- Option 3: Reflection

3	2	1	1	2	3	4	5	5	4	3	2
			7	10	16	22	27				

Why one image is reflected in the convolution:

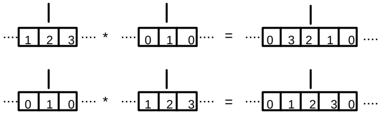
With reflection:



…. 1 2 3 …. * …. 0 1 0 …. = …. 0 3 2 1 0 ….

…. 0 1 0 …. * …. 1 2 3 …. = …. 0 3 2 1 0 ….

Without reflection:



…. 1 2 3 …. * …. 0 1 0 …. = …. 0 3 2 1 0 ….

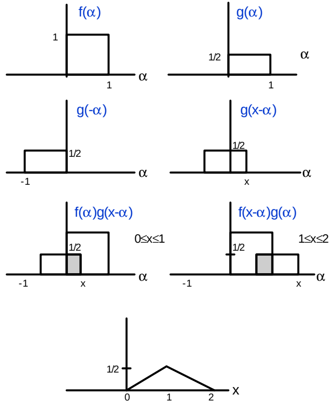
…. 0 1 0 …. * …. 1 2 3 …. = …. 0 1 2 3 0 ….

Reflection is needed so that convolution is commutative:

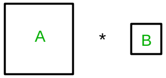
A * B = B * A

Convolution - Continuous Case

$$(f*g)(x)=\int_{-\infty}^{\infty} f(a)\,g(x-a)\,da$$



Convolution - 2 Dimensions



$$(A * B)(x,y) = \sum_i \sum_j A(i,j)\,B(x-i,y-j)$$

10	5	20	20	20
10	5	20	20	20
10	5	20	20	20
10	5	20	20	20
10	5	20	20	20

*

-1	1
0	1

=

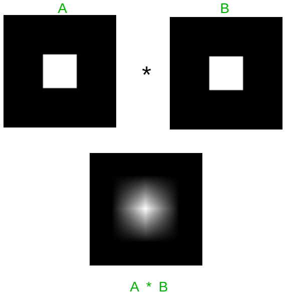
-10	5	-15	0	0
-10	15	-10	20	20
-10	15	-10	20	20
-10	15	-10	20	20
-10	15	-10	20	20

(zero padding)

Convolution -2D Continuous Case:

$$(f*g)(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,ß)\,g(x-a,y-ß)\,da\,dß$$

Grayscale Convolution - Example



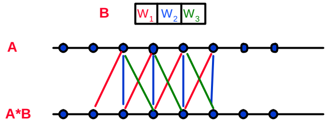
Convolution Properties

- Complexity:
  - Assume A is n x n and B is k x k then A*B takes $O(n^2k^2)$ operations.
- A*B = B*A
- (A*B)*C = A*(B*C)
  - If B and C are k x k then (A*B)*C takes $O(2n^2k^2)$ operations. However A*(B*C) takes $O(k^4+n^2k^2)$ operations, which is faster if k<<n .
- Separability
  - In some cases it is possible to decompose B (k x k) into B=C*D where C is 1 x k and D is k x 1. In such a case A*B takes $O(n^2k^2)$ while (A*C)*D takes $O(2n^2k)$.

$$\begin{array}{|c|c|}\hline 1 & -1 \\ \hline\end{array} * \begin{array}{|c|}\hline 1 \\ \hline -1 \\ \hline\end{array} = \begin{array}{|c|c|}\hline 1 & -1 \\ \hline -1 & 1 \\ \hline\end{array}$$

Mask Constraints

- Image Average
  - In order to preserve the overall average of A, the sum of B's elements should equal 1



If $W_1+W_2+W_3=1$ then Av(A)=Av(A*B)

$$d(x-x_0,y-y_0)=\begin{cases}1 & \text{if } x=x_0 \text{ and } y=y_0 \\ 0 & \text{otherwise}\end{cases}$$

$\delta(x, y)$

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array}$$

$$\boxed{A(x,y)*\delta(x-x_0,y-y_0) = A(x-x_0,y-y_0)}$$

Convolution Masks - Example: The Delta Kernel

$$\boxed{A(x,y)*\delta(x-x_0,y-y_0) = A(x-x_0,y-y_0)}$$

$$d(x-x_0,y-y_0)=\begin{cases}1 & \text{if } x=x_0 \text{ and } y=y_0\\ 0 & \text{otherwise}\end{cases}$$

$$\boxed{A(x,y)*\delta(x,y) = A(x,y)}$$

$$\underset{d(x,y)}{\begin{array}{|c|c|c|}\hline 0&0&0\\\hline 0&1&0\\\hline 0&0&0\\\hline\end{array}} \quad \underset{d(x-1,y-1)}{\begin{array}{|c|c|c|}\hline 0&0&1\\\hline 0&0&0\\\hline 0&0&0\\\hline\end{array}}$$

$$\underset{A}{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline 7&8&9\\\hline\end{array}} * \underset{d(x-1,y-1)}{\begin{array}{|c|c|c|}\hline 0&0&1\\\hline 0&0&0\\\hline 0&0&0\\\hline\end{array}} = \underset{A(x-1,y-1)}{\begin{array}{|c|c|c|}\hline 0&4&5\\\hline 0&7&8\\\hline 0&0&0\\\hline\end{array}} \quad \text{(Zero padding)}$$

$$= \underset{A(x-1,y-1)}{\begin{array}{|c|c|c|}\hline 6&4&5\\\hline 9&7&8\\\hline 3&1&2\\\hline\end{array}} \quad \text{(Wrap around)}$$

Grayscale Smoothing

Grayscale averaging = convolution with:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3 X 3

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

5 X 5

"Soft" Averaging: Convolution with a Gaussian

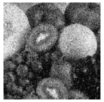
$$\frac{1}{2ps^{2}}e^{\frac{-(x^{2}+y^{2})}{2s^{2}}}$$

Discrete case:

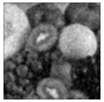
$$(1/8)\ \text{x} \begin{array}{|c|c|c|}\hline 0&1&0\\\hline 1&2&1\\\hline 0&1&0\\\hline\end{array} \qquad (1/81)\ \text{x} \begin{array}{|c|c|c|c|c|}\hline 1&2&3&2&1\\\hline 2&4&6&4&2\\\hline 3&6&9&6&3\\\hline 2&4&6&4&2\\\hline 1&2&3&2&1\\\hline\end{array}$$

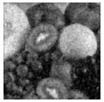
A seperable kernel

Normal vs Gaussian Grayscale Smoothing


Original
Noisy image

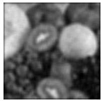
3 X 3
Average

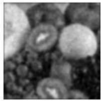




3 X 3
Gaussian
Average

5 X 5
Average





5 X 5
Gaussian
Average

7 X 7
Average

7 X 7
Gaussian
Average

Median Filtering

S = neighborhood of pixel (x,y)

New value at (x,y) = $\underset{(x,y)\in S}{\text{median}} \{I(x,y)\}$

30	10	20
10	250	25
20	25	30

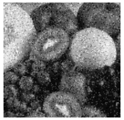
$\longrightarrow$ 10, 10, 20, 20, 25, 25, 30, 30, 250

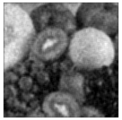
median

Median + Average: average the k central values.

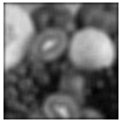
$10, 10, \underbrace{20, 20, 25, 25, 30, 30}_{24}, 250$

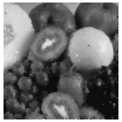
Median vs Average Filtering


Salt & Pepper Noise

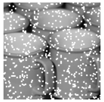

3 X 3 Average


5 X 5 Average


7 X 7 Average


Median

Multiple Median Filtering


Large Noise


Median


Median x 2

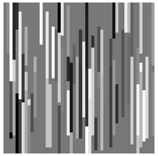

Median x 4

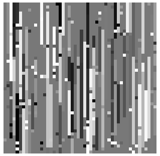

Median x 8

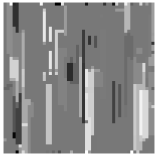

Median x 6


Median x 7

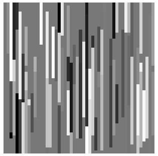
Median Filtering - Failure

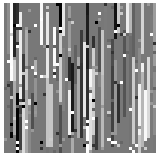

Original

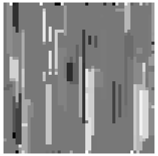

Salt & Pepper Noise

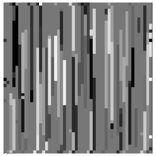

Median Filter

Oriented Median Filtering


Original


Salt & Pepper Noise

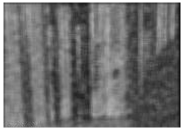

Median Filter


Oriented Median Filter

Oriented Filters

Salt & Pepper noise

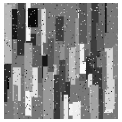
4x4 Average

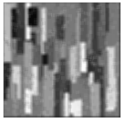


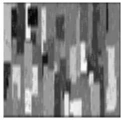
7x2 Average

Oriented Filtering - Example


Original

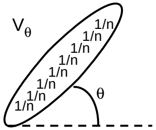

Noisy Image


4x4 Average


Oriented 6x2 Average

Directional Smoothing

Define oriented masks:



Choose neighborhood with smallest variance and replace pixel value with the average of that neighborhood.

Directional Smoothing - Example

Original + Noise

3x3 Average


Directional Smoothing (2x5, 5x2, diagonalx2)

Directional Smoothing - Example

Original + Noise

3x3 Average

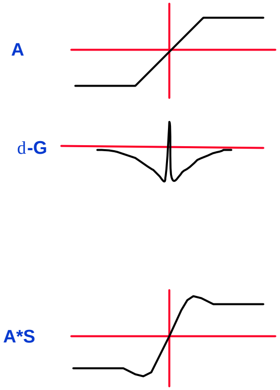
Directional Smoothing
(2x5, 5x2, diagonalx2)

Sharpening

- A sharpening filter is applied in order to enhance edges and fine details (high frequency) in an image:
  - Assume A*G is a smoothing filtering.
  - A*(δ - G) contains the fine details of the image.
  - A+λA*(δ - G) = A*((1+λ)δ- λG)=A*S(λ) amplifies fine details in the image.
  - The parameter λ controls the amount of amplification.

$$G = \begin{pmatrix} 0 & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{2} & \frac{1}{8} \\ 0 & \frac{1}{8} & 0 \end{pmatrix} \quad S(1) = \begin{pmatrix} 0 & -\frac{1}{8} & 0 \\ -\frac{1}{8} & \frac{3}{2} & -\frac{1}{8} \\ 0 & -\frac{1}{8} & 0 \end{pmatrix}$$

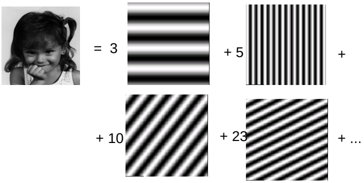
Ringing effect in edge enhancement



How can we enhance such an image?

Solution: Image Representation

$$\begin{bmatrix}2&1&3\\5&8&7\\0&3&5\end{bmatrix} = 2\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix} + 1\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix} + 3\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix} + 5\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix} + \ldots$$



Frequency Domain

Map of “Sizes and Orientations”

Evaluating an Image in terms of ‘sizes’: