Image Processing - Lesson 2

## Binary Images (Part I)

- Threshold
- Binary Image - Definition
- Connected Components
- Chain Code
- Edge Following

Grayscale Image
Binary Image


## Thresholding



Threshold

## Thresholding a Grayscale Image

Original Image


Threshold too low


Binary Image


Threshold too high


## FMRI - Example

Original Image


Threshold = 71

Threshold $=80$


Threshold = 88

## Segmentation using Thresholding

Original


Histogram



Threshold $=50$


Threshold $=75$

Original



Threshold = 21

(Edgar Rubin 1915)

## Connected Components



## Neighborhoods:



4-neighbor metric


8-neighbor metric

## Connected Components:

$S=$ the set of object pixels
$S$ is a Connected Component if for each pixel pair $\left(x_{1}, y_{1}\right) \in S$ and $\left(\mathrm{X}_{2}, \mathrm{y}_{2}\right) \in \mathrm{S}$ there is a path passing through X -neighbors in $S$. $(X=4,8)$.

S may contain several connected components.


## 1 connected component-8

## 3 connected components-4



8-neighborhood:
1 object connected component
1 background connected component
4-neighborhood:
2 background connected components
4 object connected components

Always choose different neighborhood metrics for objects and backgrounds.

## Chain Code

Each direction is assigned a code:


$$
3
$$

4-neighbor


4-neighbor:


8-neighbor

|  |  |  | $\bullet$ | $\bullet$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | $\bullet$ |  | $\bullet$ |  |
|  |  | $\bullet$ |  |  | $\bullet$ |  |
|  |  | $\bullet$ |  |  | $\bullet$ |  |
|  | $\bullet$ |  |  |  | $\bullet$ |  |
|  |  | $\bullet$ |  | $\bullet$ |  |  |
|  |  |  |  |  | $\bullet$ |  |

8-neighbor:


## Marking the Connected Components

Connected Component Algorithm: Two passes over the image.

## Pass 1:

Scan the image pixels from left to right and from top to bottom. For every pixel P of value 1 (an object pixel), test top and left neighbors (4-neighbor metric).

- If 2 of the neighbors are 0 : assign a new mark to $P$.
- If 1 of the neighbors isn't 0 : assign the neighbor's mark to $P$.
- If 2 of the neighbors are not 0 : assign the left neighbor's mark to $P$


## Pass 2 :

Divide all marks to equivalence classes (marks of neighboring pixels are considered equivalent).
Replace each mark with the number of its equivalence class.


## Connected Components - Example

Original Binary image

Pass 1:

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 1 | 1 |  |  |  |  |
| 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |
|  |  |  |  |  |  |  |  |


|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 1 | 1 |  |  |  |  |
| 2 | 2 | 2 | 2 |  | 3 | 3 | 3 |
|  |  |  |  |  |  |  |  |

Pass 2:

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 1 | 1 |  |  |  |  |
| 1 | 1 | 1 | 1 |  | 2 | 2 | 2 |
|  |  |  |  |  |  |  |  |

Equivalence Class number

Original mark

1,2
3

## Connected Components - Example II

Original Binary image

Pass 1:


|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | 1 |  |  |  |  |
|  |  | 2 | 2 |  | 3 | 3 |  |
|  | 4 | 4 | 4 | 4 | 4 |  |  |


|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | 1 |  |  |  |  |
|  |  | 1 | 1 |  | 1 | 1 |  |
|  | 1 | 1 | 1 | 1 | 1 |  |  |

Equivalence Class number

Original mark
1,2,3,4

## Edges

$$
\begin{aligned}
& C=\text { connected component of object } S . \\
& D=\text { connected component of } \bar{S} .
\end{aligned}
$$

The $D$-Edge of $C=$ the set of all pixels in $C$ that have $a$ neighboring pixel in D . (neighboring-8 if $C$ is 4-connected neighboring-4 if C is 8 -connected).

## Example:



(4-neighbor)

The Edge of C for background D.

The Edge of C for hole D.

## Distances

Two grid point: $\quad P=(x, y) \quad$ and $\quad Q=(u, v)$

Euclidean Distance

$$
d_{e}(P, Q)=\sqrt{(x-u)^{2}+(y-v)^{2}}
$$

City Block Distance

$$
d_{4}(P, Q)=|x-u|+|y-v|
$$

Chessboard Distance

$$
d_{8}(P, Q)=\max (|x-u|,|y-v|)
$$

$$
\begin{aligned}
& d_{e}=7.6 \\
& d_{8}=7 \\
& d_{4}=10
\end{aligned}
$$



## $d_{e} d_{8} d_{4}$ are all metrics:

1. Distance metric: $\quad d(P, Q) \geq 0$
2. Positive:

$$
\begin{aligned}
& d(P, Q)=0 \text { iff } P=Q \\
& d(P, Q)=d(Q, P)
\end{aligned}
$$

4. Triangular inequality: $d(P, Q) \leq d(P, R)+d(R, Q)$

All pixels at equal $d_{4}$ distance form a "diamond" :


All pixels at equal $\mathrm{d}_{8}$ distance form a "square" :

| 2 | 2 | 2 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | 0 | 1 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 |

All pixels at equal $d_{e}$ distance form a "circle" :


## 2-Pass Distance Algorithm

For each pixel calculate the $d_{4}$ or $d_{8}$ distance from a pixel in set $S$.
2 passes:
Pass 1: scan image left-to-right and top-to-bottom
Pass 2: scan image right-to-left and bottom-to-top.
For each pixel P mark as follows:


Pass 1: consider all neighbors of $P$ that have been scanned $N_{1}=\square$

$$
d^{\prime}(P, S)= \begin{cases}0 & \text { if } P \in S \\ \min \left\{d^{\prime}(Q, S)\right\}+1 & \text { if } P \notin S \\ Q \in N_{1} & \end{cases}
$$

Pass 2: consider all neighbors of $P$ that have been scanned $N_{2}=\square$

$$
d^{\prime \prime}(P, S)=\min _{Q \in N_{2}}\left\{d^{\prime}(P, S), d^{\prime \prime}(Q, S)+1\right\}
$$

Example measuring $d_{4}$ :

| 1 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 2 | 3 | 0 | 1 | 2 | 1 | 0 |
| 0 | 0 | 0 | 0 | 2 | 3 | 4 | 1 | 2 | 3 | 2 | 1 |

S is marked as $1 \quad$ Pass 1: d' $(P, S) \quad$ Pass 2: d" $(P, S)$

## Skeletons

Consider all edge pixels of an object as the group $S$.


The pixels whose distance is a local maxima are the Skeleton of the object.

The Skeleton can be used as a shape descriptor.

## MAT = Medial Axis Transform

Grass fire technique (Blum, 1993)


## Skeletons - Example



Sensitivity to contour changes:


Distance Map


Skeleton

