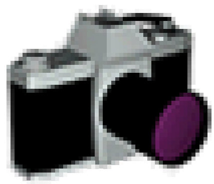
Image Acquisition + Histograms

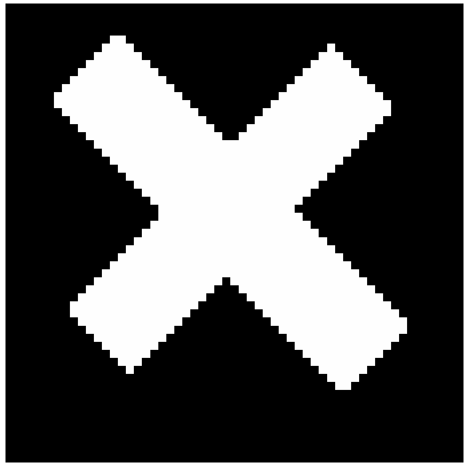
- Image Characteristics
- Image Acquisition
- Image Digitization
    - Sampling
    - Quantization
- Histograms
- Histogram Equalization

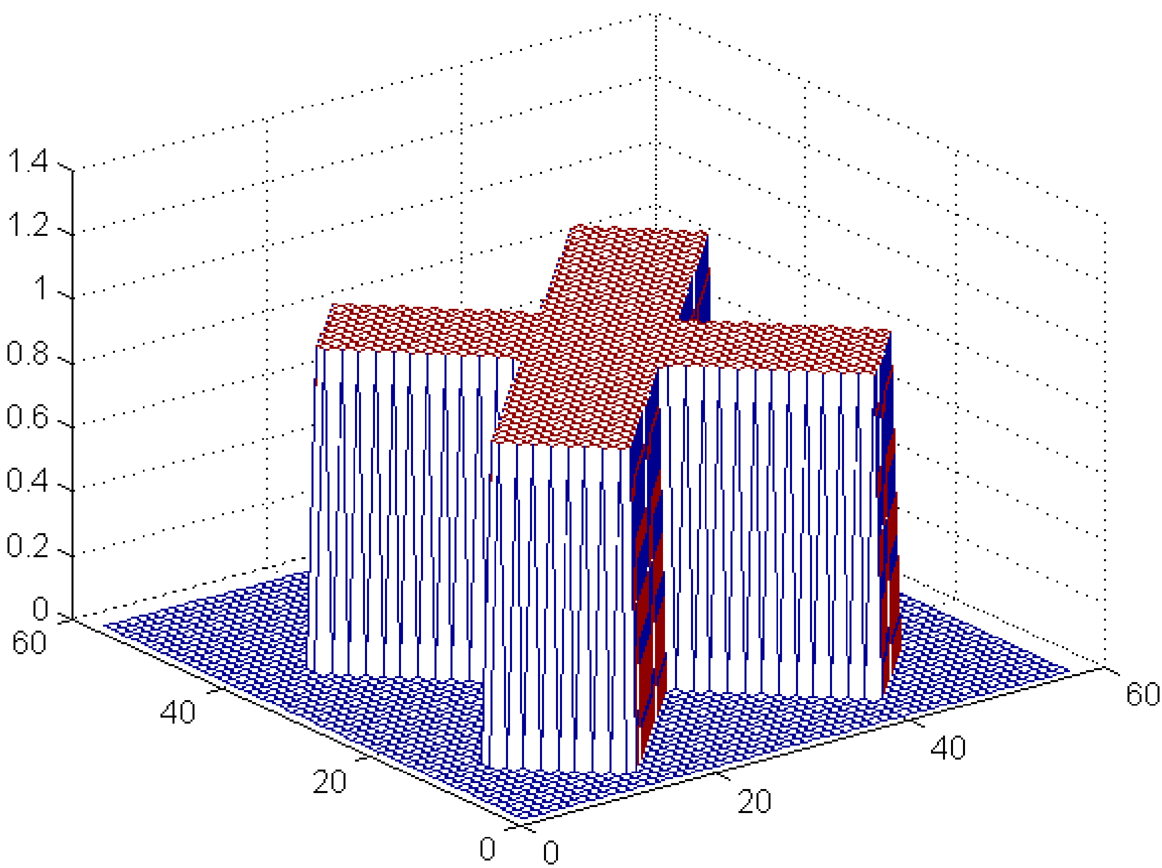
What is an Image ?

- An image is a projection of a 3D scene into a 2D *projection plane*.


- An image can be defined as a 2 variable function I(x,y) , where for each position (x,y) in the projection plane, I(x,y) defines the light intensity at this point.





1.4
1.2
1
0.8
0.6
0.4
0.2
0
60
40
20
0
0
20
40
60

- Three types of images:
  - Binary images

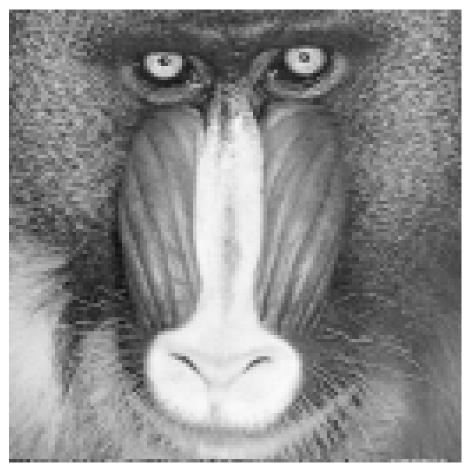
$$I(x,y) \in \{0 , 1\}$$

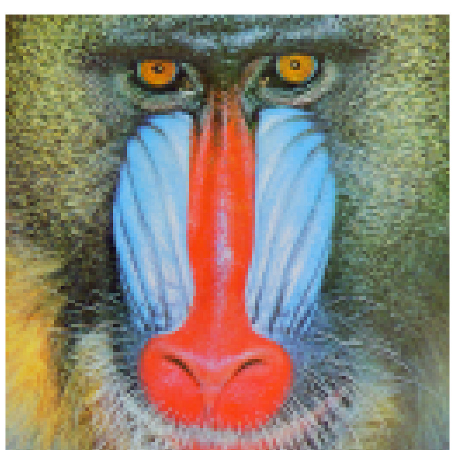
  - Gray-scale images

$$I(x,y) \in [a , b]$$

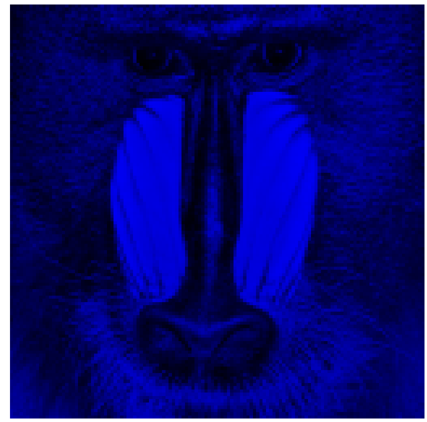
  - Color Images

$$I_R(x,y) \quad I_G(x,y) \quad I_B(x,y)$$









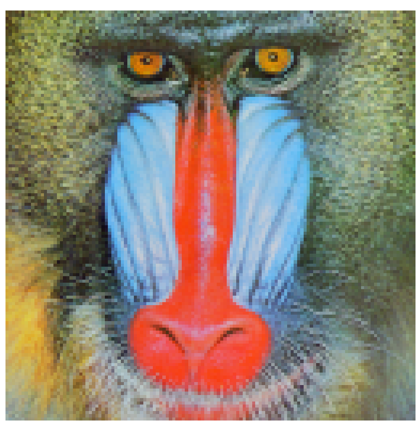


Image Values

- Image Intensity -
  - Light energy emitted from a unit area in the image.
  - Device dependence.

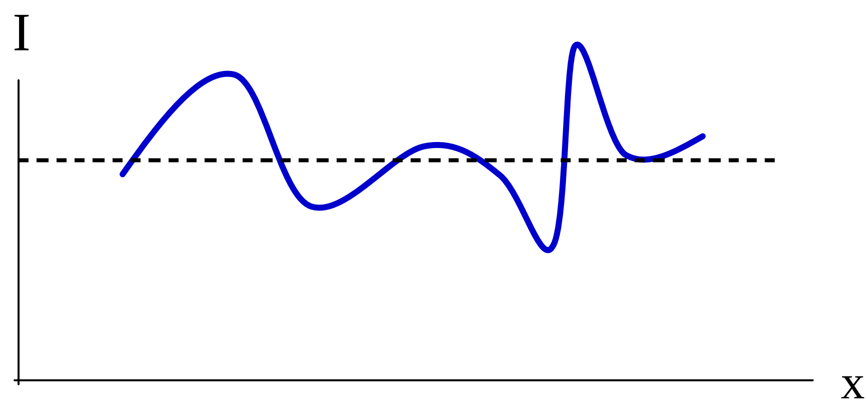
- Image Brightness -
  - The subjective appearance of a unit area in the image.
  - Context dependence.
  - Subjective.

- Image Gray-Level -
  - The relative intensity at each unit area.
  - Between the lowest intensity (Black value) and the highest intensity (White value).
  - Typical: In the range of [0,1] or [0,255]

Image Average (Brightness)

- Image average:

$$I_{av} = \frac{\int_y \int_x I(x,y)\,dx\,dy}{\int_y \int_x dx\,dy}$$



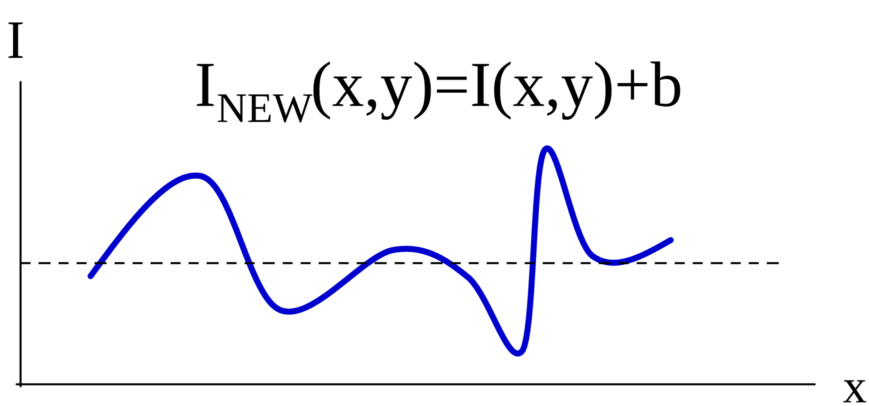
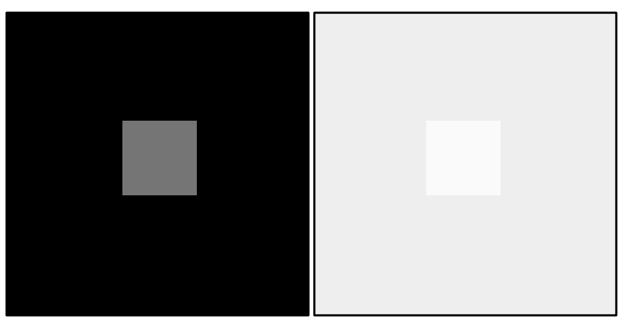




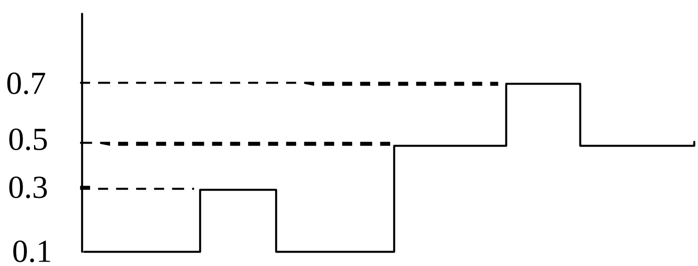
Image Contrast

- The contrast at an image point denotes the (relative) difference between the intensity of the point and the intensity of its neighborhood:

$$C = \left| \frac{I_p - I_n}{I_n} \right|$$



$C = \left| \frac{0.3 - 0.1}{0.1} \right| = 2$ $\qquad$ $C = \left| \frac{0.7 - 0.5}{0.5} \right| = 0.4$

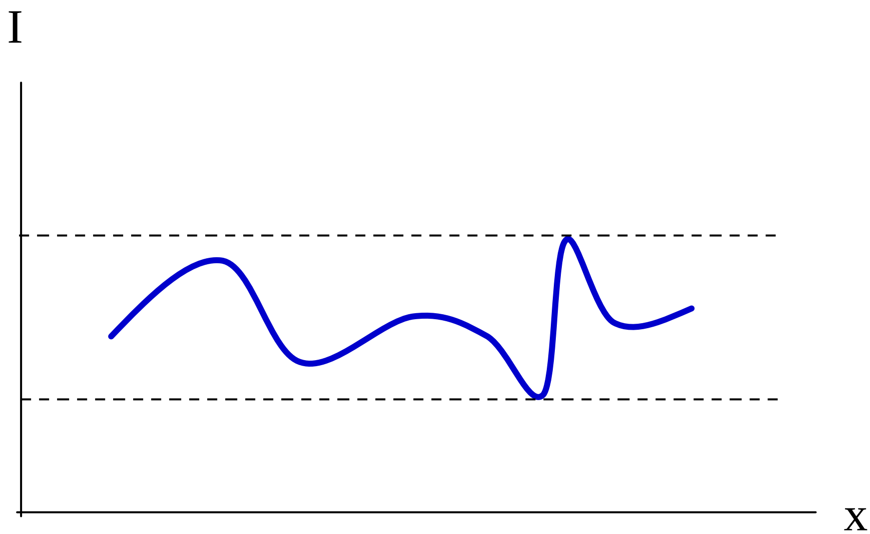


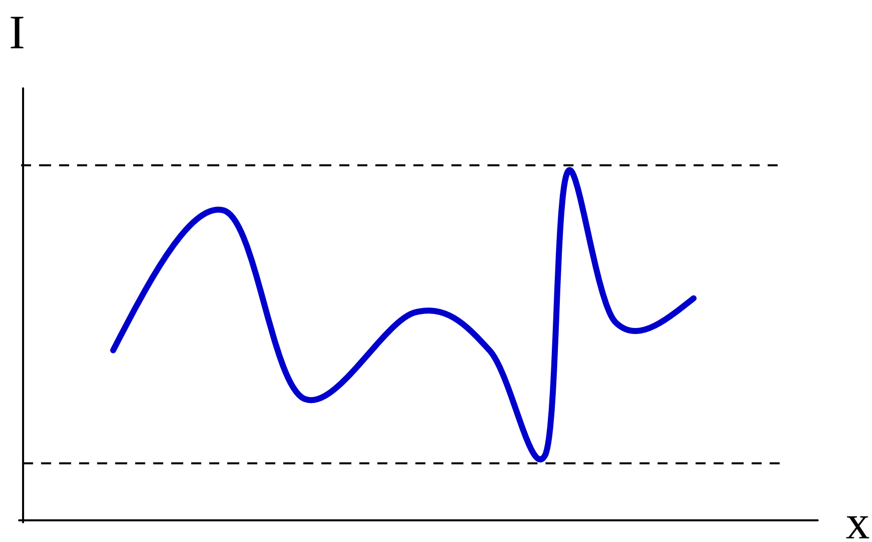
- The contrast definition of the entire image is ambiguous.
- In general it is said that the image contrast is high if the image gray-levels fill the entire range.

Low contrast

High contrast

- Contrast manipulation of the entire image can be done by *stretching* and *shifting* the image gray-levels

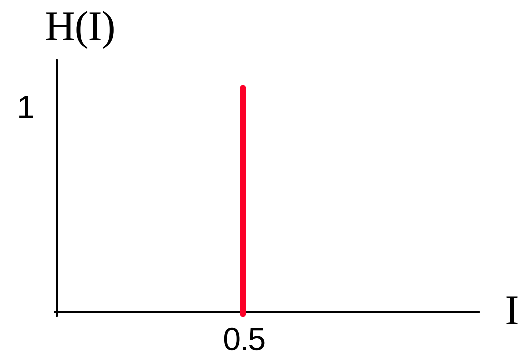


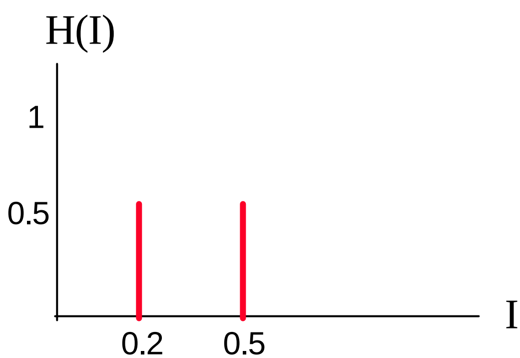


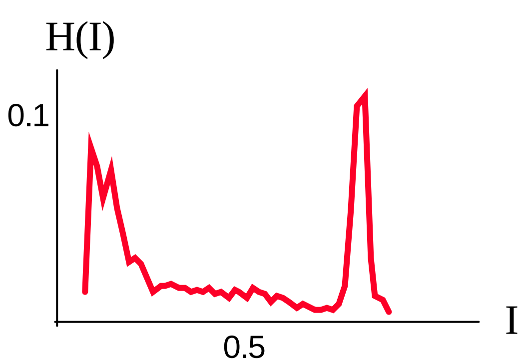
$I_{NEW}(x,y)=a*(I(x,y)-b)+c$

The Image Histogram

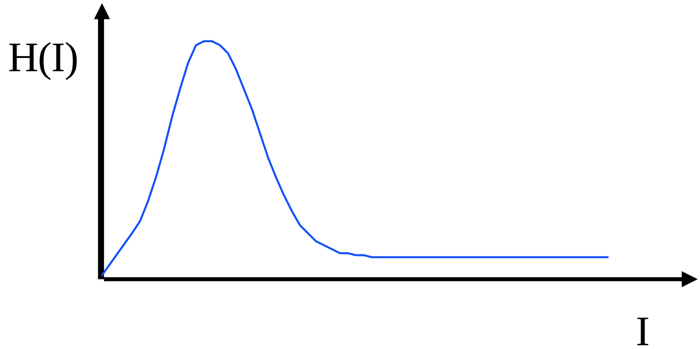
- Image Histogram describes the relative proportion of gray-level *I* in the image plane:

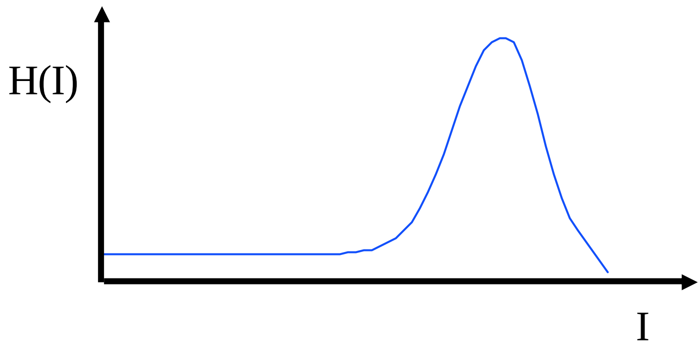


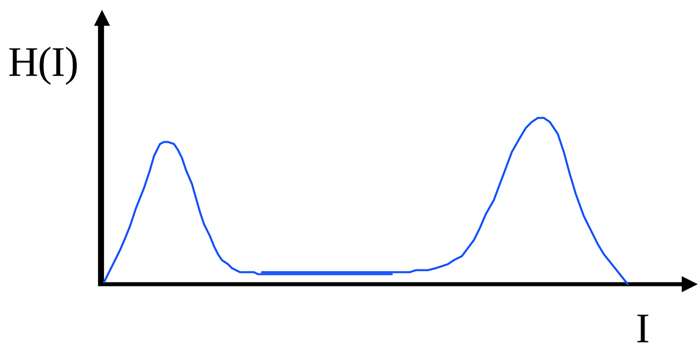


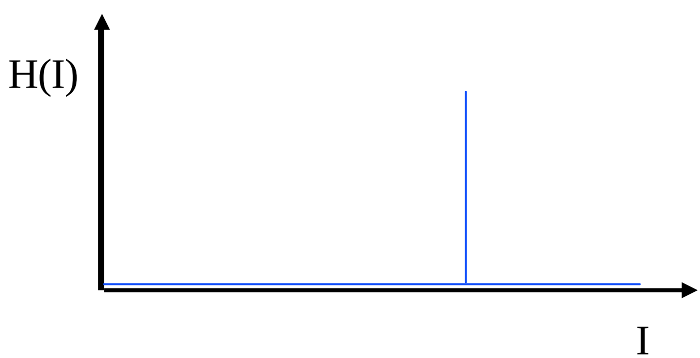


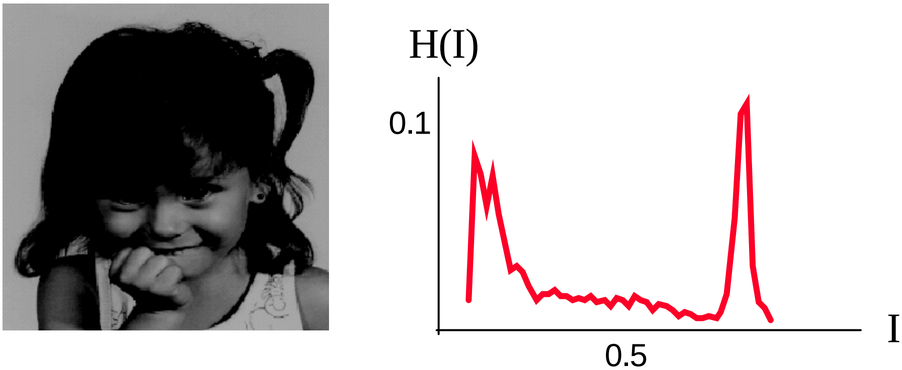
Histograms - Examples



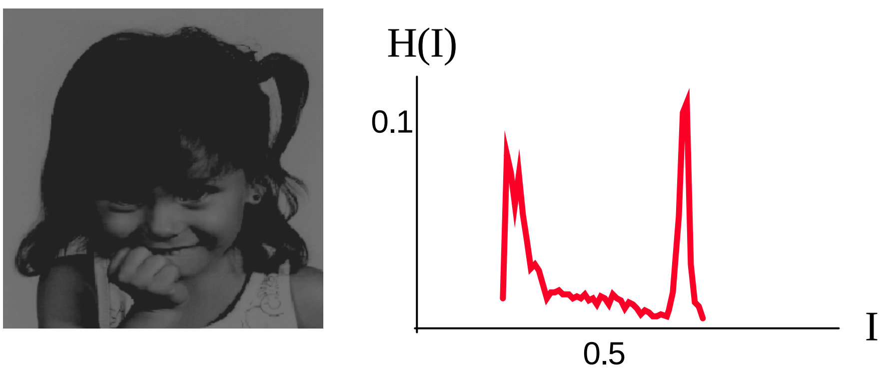








- Decreasing the image contrast:



- Increasing the new image average :

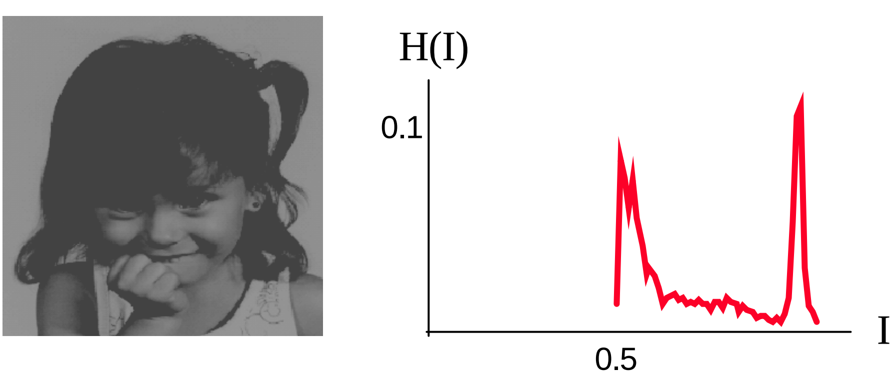
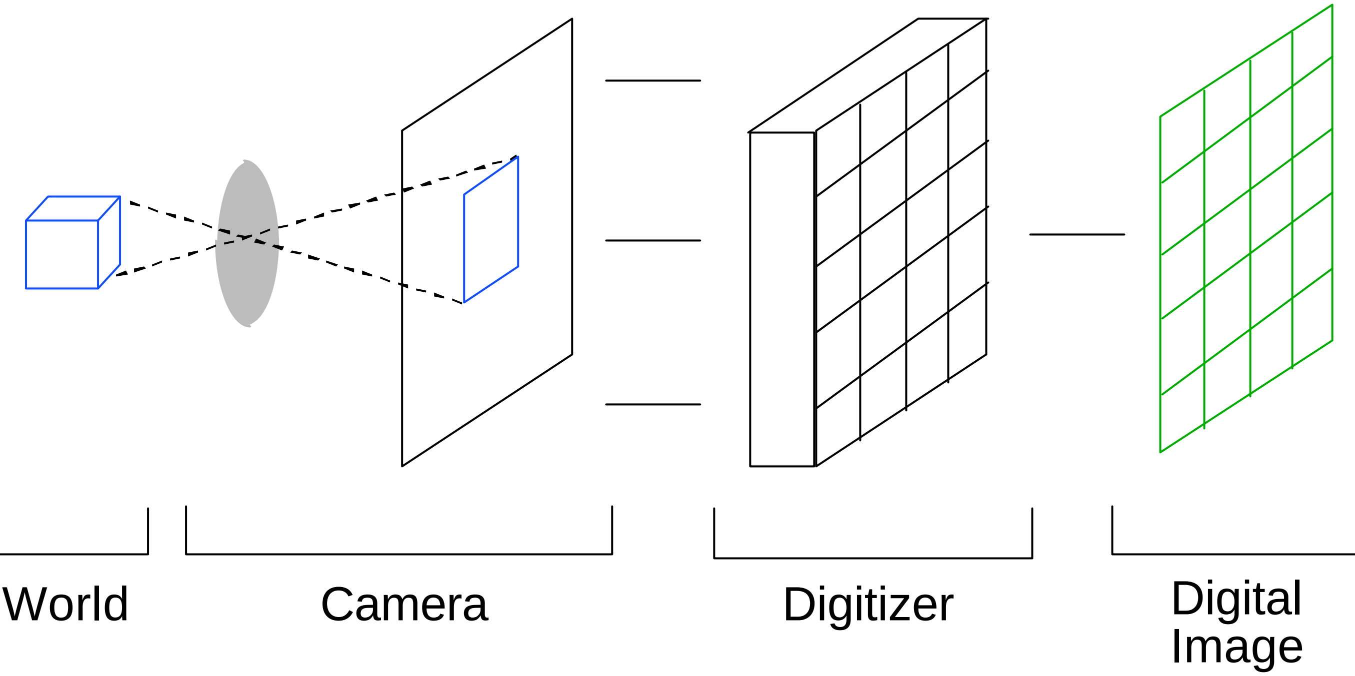


Image Acquisition



0	10	10	15	50	70	80
0	0	100	120	125	130	130
0	35	100	150	150	80	50
0	15	70	100	10	20	20
0	15	70	0	0	0	15
5	15	50	120	110	130	110
5	10	20	50	50	20	250

PIXEL
(picture element)

Typically:
0 = black
255 = white

Digitization

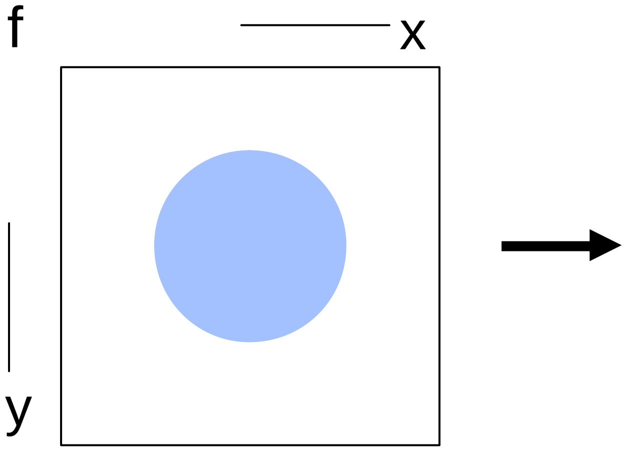


Image Plane

	1	2	3	4	5
1	100	100	100	100	100
2	100	0	0	0	100
3	100	0	0	0	100
4	100	0	0	0	100
5	100	100	100	100	100

Digital Image

Continuous function of brightness:

$f(x,y) = i_{xy}$

where:

x,y R

i_{xy} R

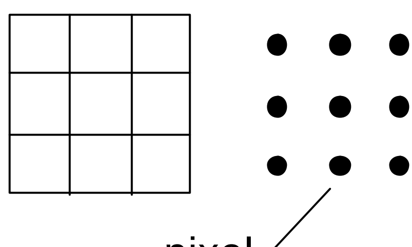
Array of gray levels:

$g_{i,j} \in K$

where:

|K| is bounded

i,j = 1,2,3,....,c



Stages in the Digitization Process:

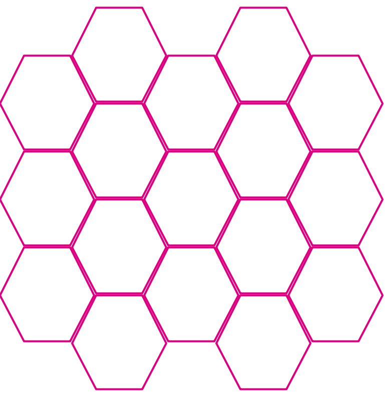
- SAMPLING - spatial
- QUANTIZATION - gray level

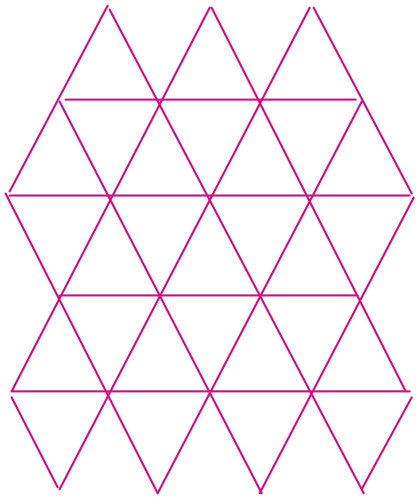
SAMPLING

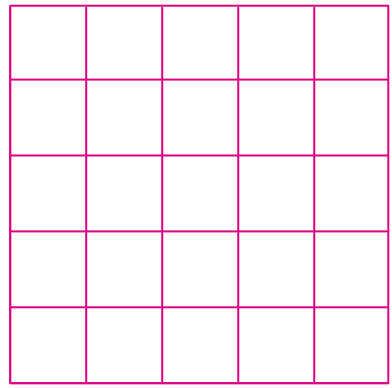
Two principles:

coverage of the image plane

uniform sampling (pixels are same size and shape)







Hexagonal -

6 neighbors
1 cell orientation
3 principle directions
non-recursive

Triangular -

neighbors: 3 edge neighbors
3 across corner
6 side corner
2 cell orientations
3 principle directions
recursive

Square Grid -

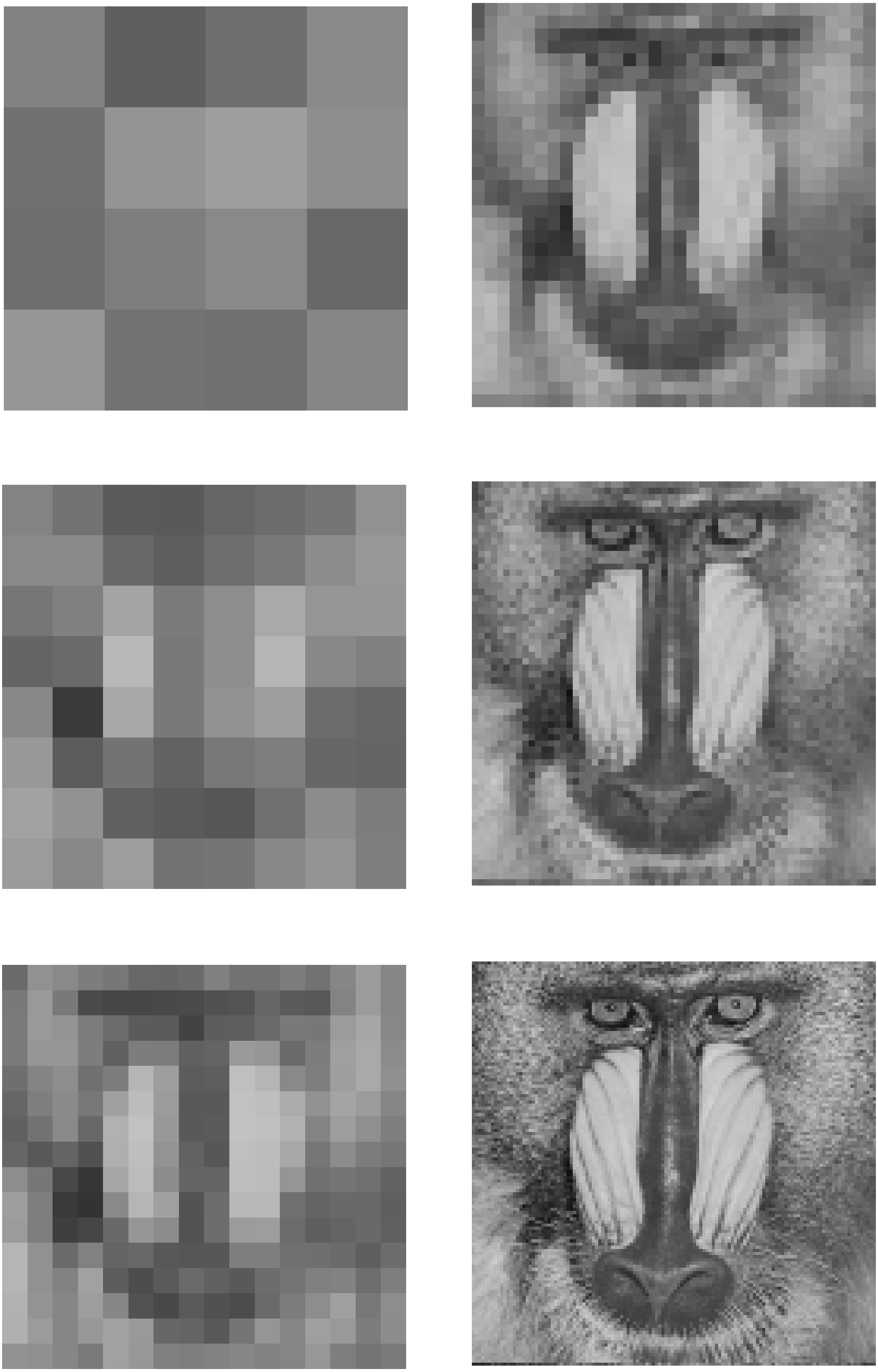
neighbors: 4 edge neighbors
4 corner neighbors
1 cell orientation
2 principle directions
recursive

Used by most equipment (Raster)

Grayscale Image

y = \ x =	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
41	210	209	204	202	197	247	143	71	64	80	84	54	54	57	58
42	206	196	203	197	195	210	207	56	63	58	53	53	61	62	51
43	201	207	192	201	198	213	156	69	65	57	55	52	53	60	50
44	216	206	211	193	202	207	208	57	69	60	55	77	49	62	61
45	221	206	211	194	196	197	220	56	63	60	55	46	97	58	106
46	209	214	224	199	194	193	204	173	64	60	59	51	62	56	48
47	204	212	213	208	191	190	191	214	60	62	66	76	51	49	55
48	214	215	215	207	208	180	172	188	69	72	55	49	56	52	56
49	209	205	214	205	204	196	187	196	86	62	66	87	57	60	48
50	208	209	205	203	202	186	174	185	149	71	63	55	55	45	56
51	207	210	211	199	217	194	183	177	209	90	62	64	52	93	52
52	208	205	209	209	197	194	183	187	187	239	58	68	61	51	56
53	204	206	203	209	195	203	188	185	183	221	75	61	58	60	60
54	200	203	199	236	188	197	183	190	183	196	122	63	58	64	66
55	205	210	202	203	199	197	196	181	173	186	105	62	57	64	63

Using Different Number of Samples



N = 4

N = 32

N = 8

N = 64

N = 16

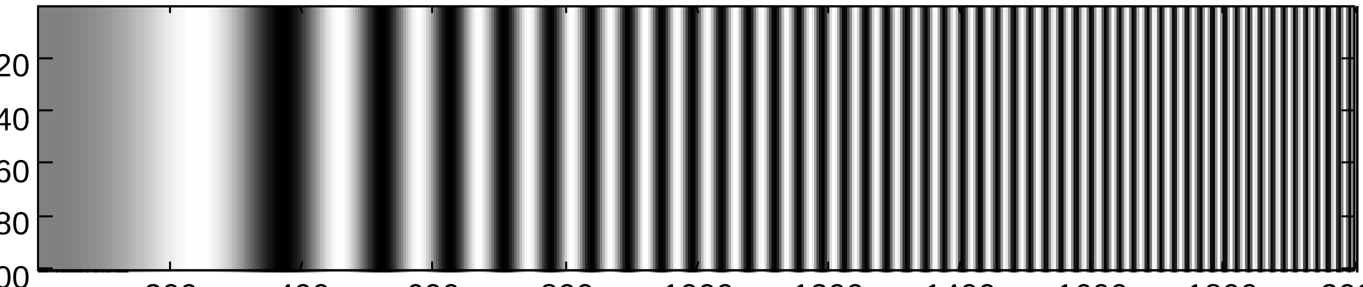
N = 128

Image Resolution

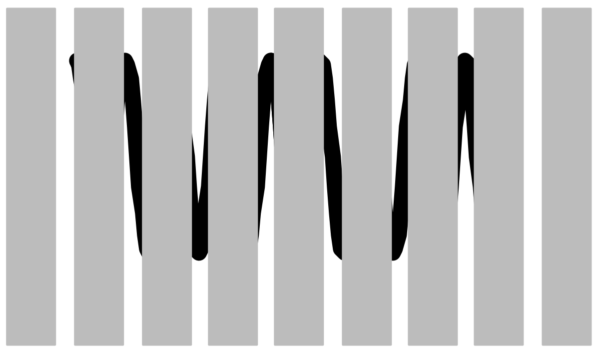
- The density of the sampling denotes the separation capability of the resulting image.
- Image resolution defines the finest details that are still visible by the image.
- We use a cyclic pattern to test the separation capability of an image.

$$\text{frequency} = \frac{\text{number of cycles}}{\text{unit length}}$$

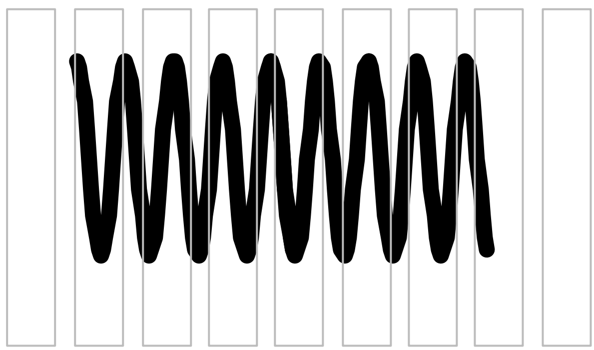
$$\text{wavelength} = \frac{\text{unit length}}{\text{number of cycles}} = \frac{1}{\text{frequency}}$$



Nyquist Frequency



Nyquist Frequency

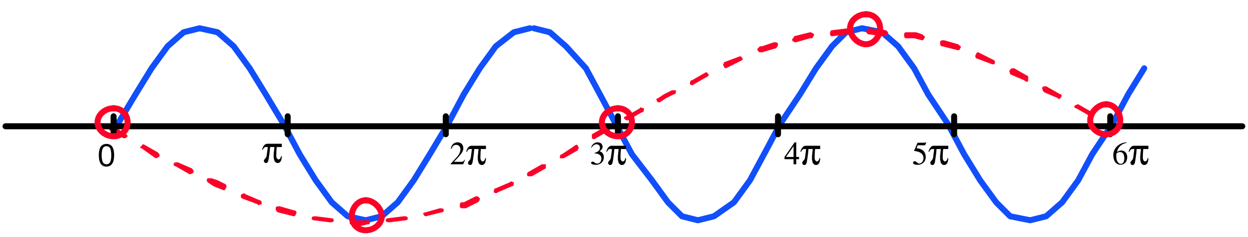


Sampling Density

• **Nyquist Rule**: Given a sampling at intervals equal to **d** then one may recover cyclic patterns of wavelength **> 2d**.
(**Shannon-Whittaker-Kotelnikov theorem**).

• **Aliasing**: If the pattern wavelength is **less** than 2**d** erroneous patterns may be produced.

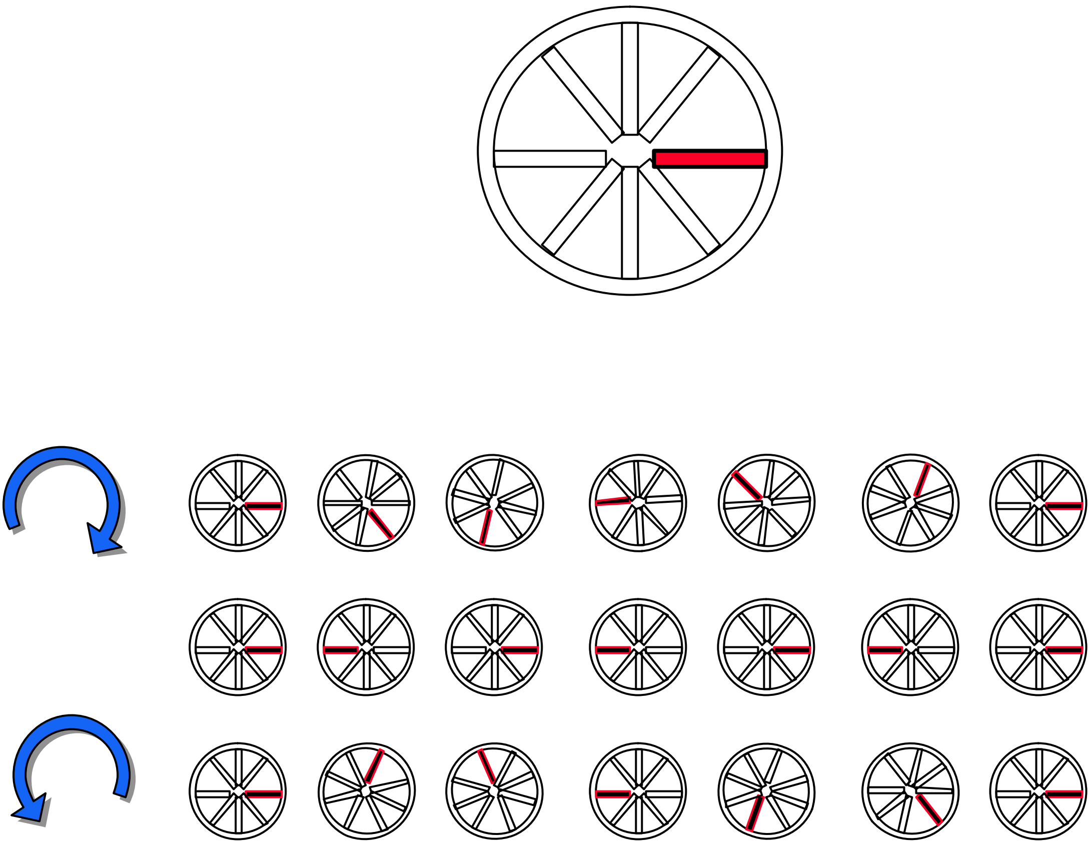
1D Example:



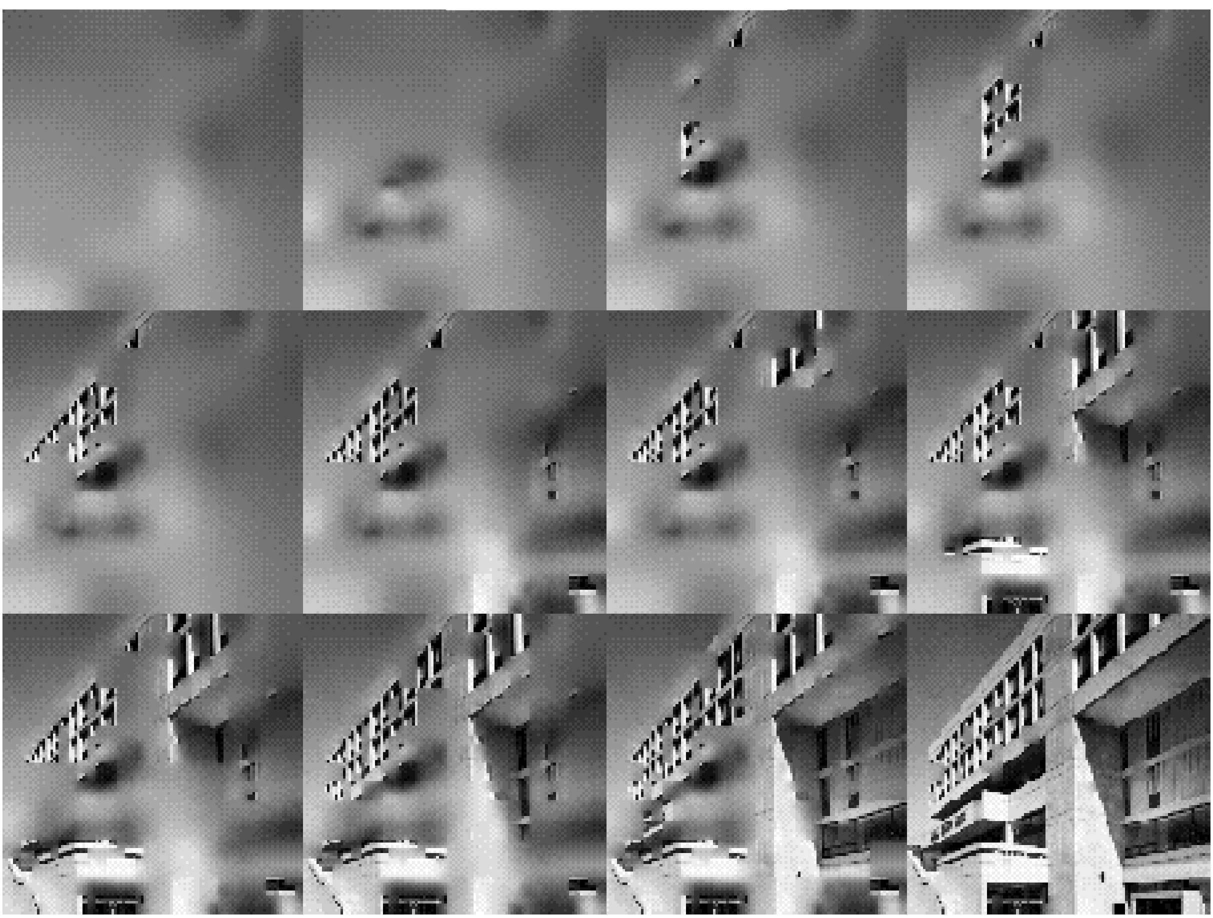
• To observe details at frequency **f** one must sample at frequency > **2f**.

• The Frequency **2f** is the **NYQUIST frequency**.

Temporal Aliasing

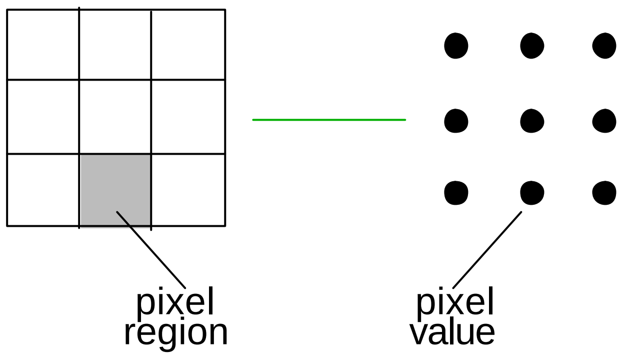


Non Uniform Sampling



Quantization

$f(x,y) = i_{xy}$ $g_{i,j} \in K$



Continuous Intensity Range — Discrete Gray Levels

- Choose number of gray levels (according to number of assigned bits).
- Divide continuous range of intensity values.

Different Number of Gray Levels

bits=1

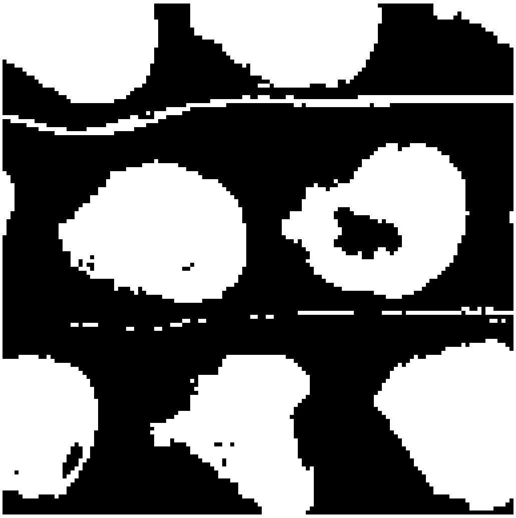
bits=2

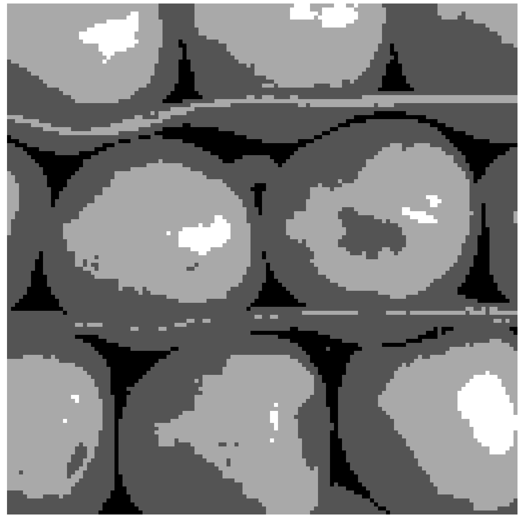
bits=3

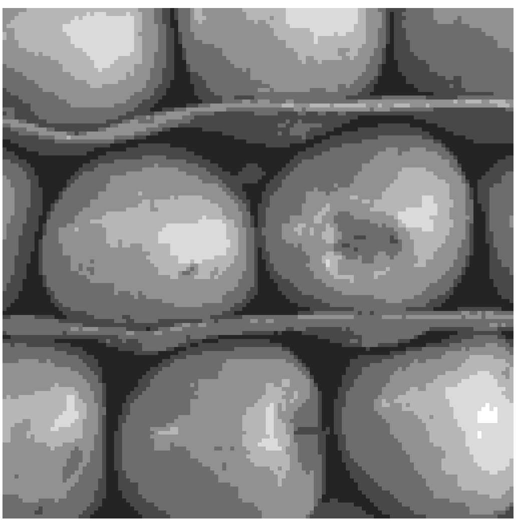
bits=4

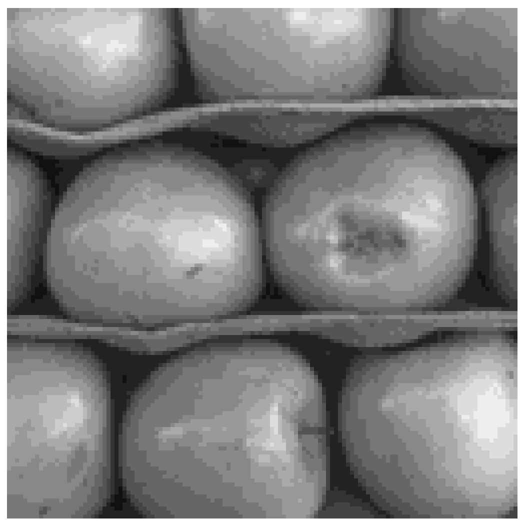
bits=8

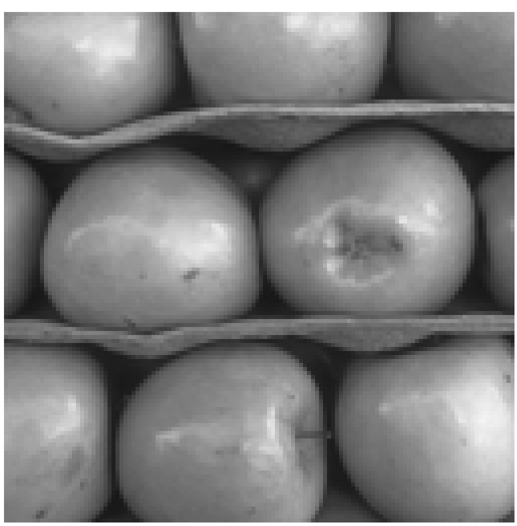
Different Number of Gray Levels


bits=1

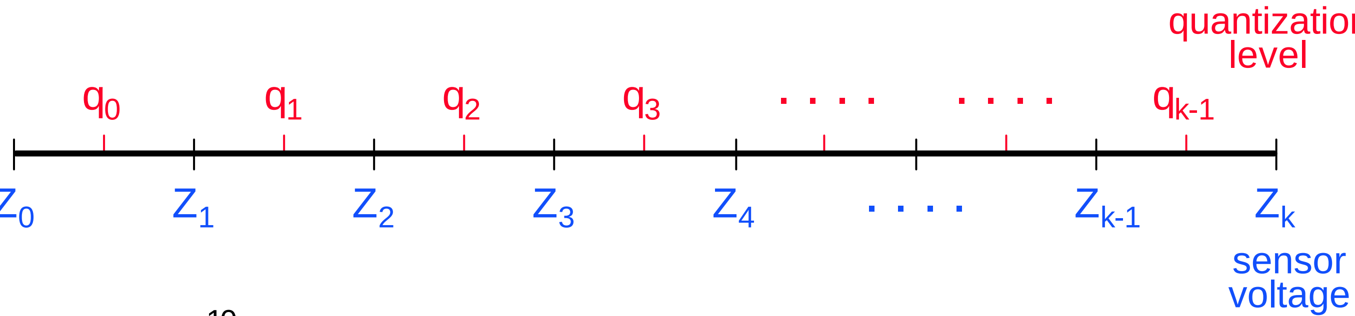

bits=2

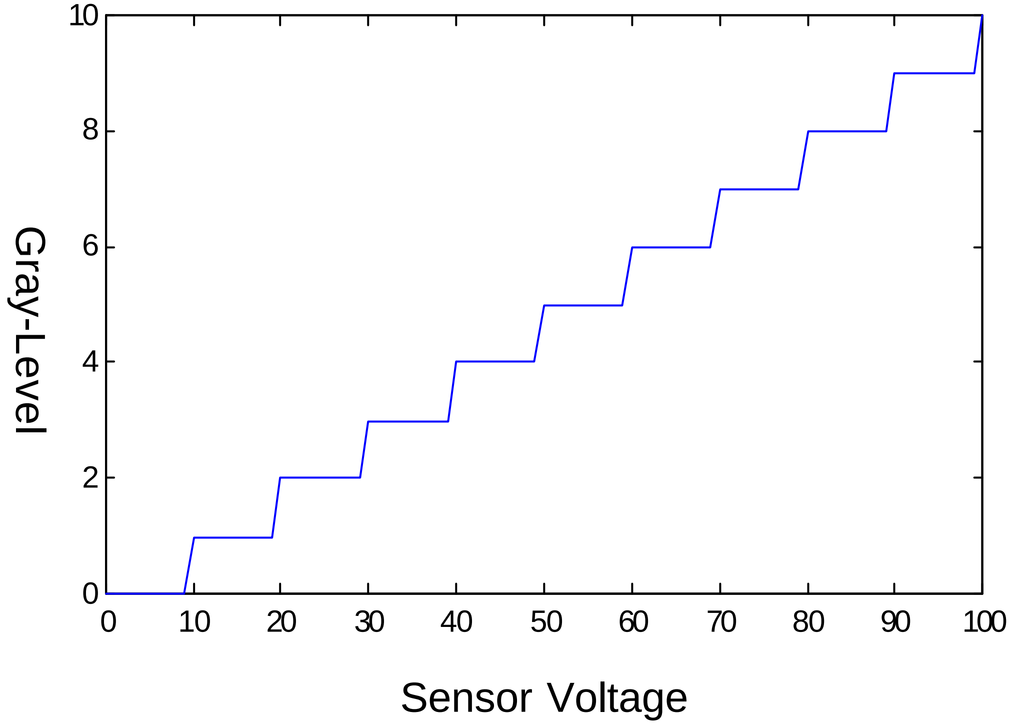

bits=3


bits=4


bits=8

Uniform Quantization:



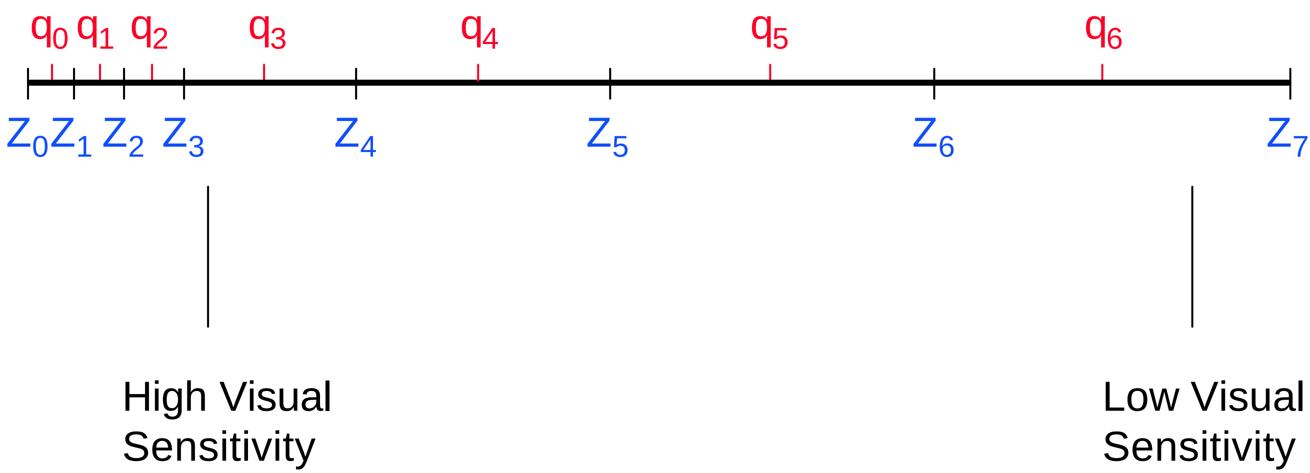


$$Z_{i+1} - Z_i = \frac{Z_k - Z_0}{K}$$

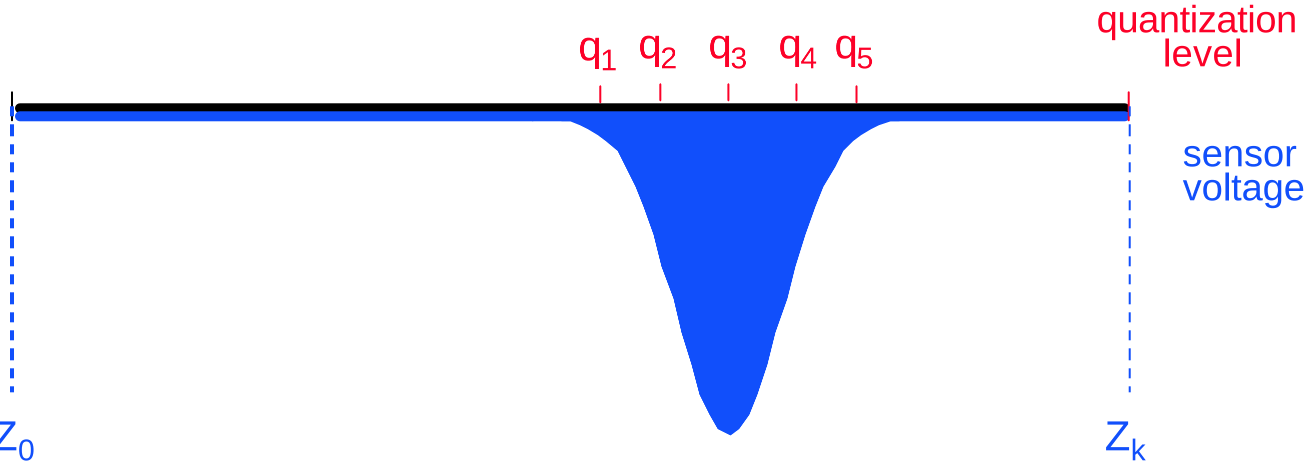
$$q_i = \frac{Z_{i+1} + Z_i}{2}$$

Non-Uniform Quantization

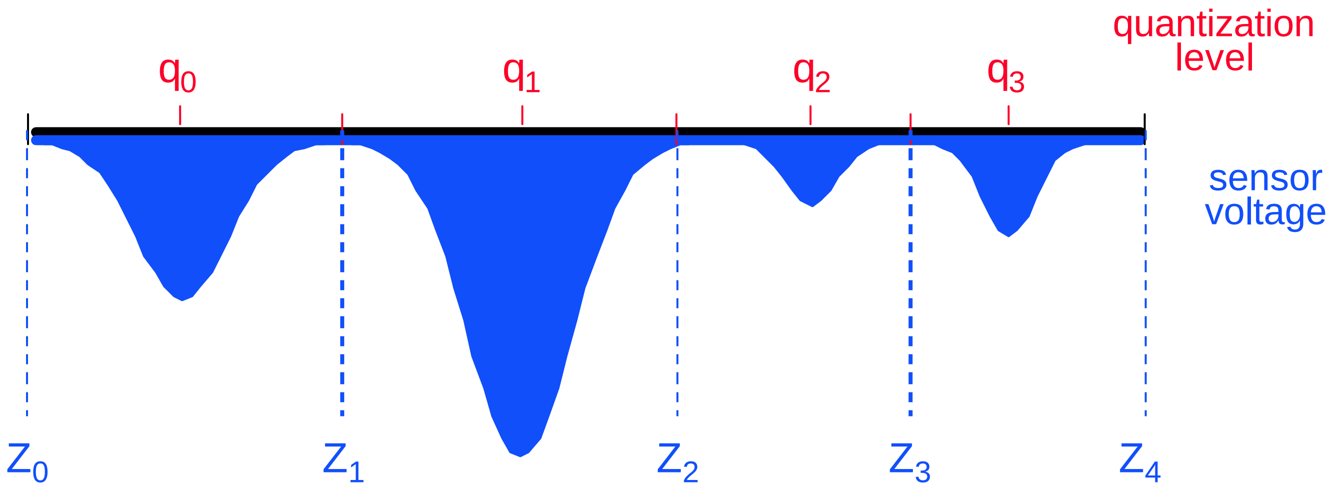
1. Non uniform visual sensitivity.



2. Non uniform sensor voltage distribution



Optimal Quantization



Minimizing the quantization error:

$$\sum_{i=0}^{k-1} \int_{Z_i}^{Z_{i+1}} (q_i - Z)^2 P(Z) dz$$

where P(Z) is the distribution of sensor voltage.

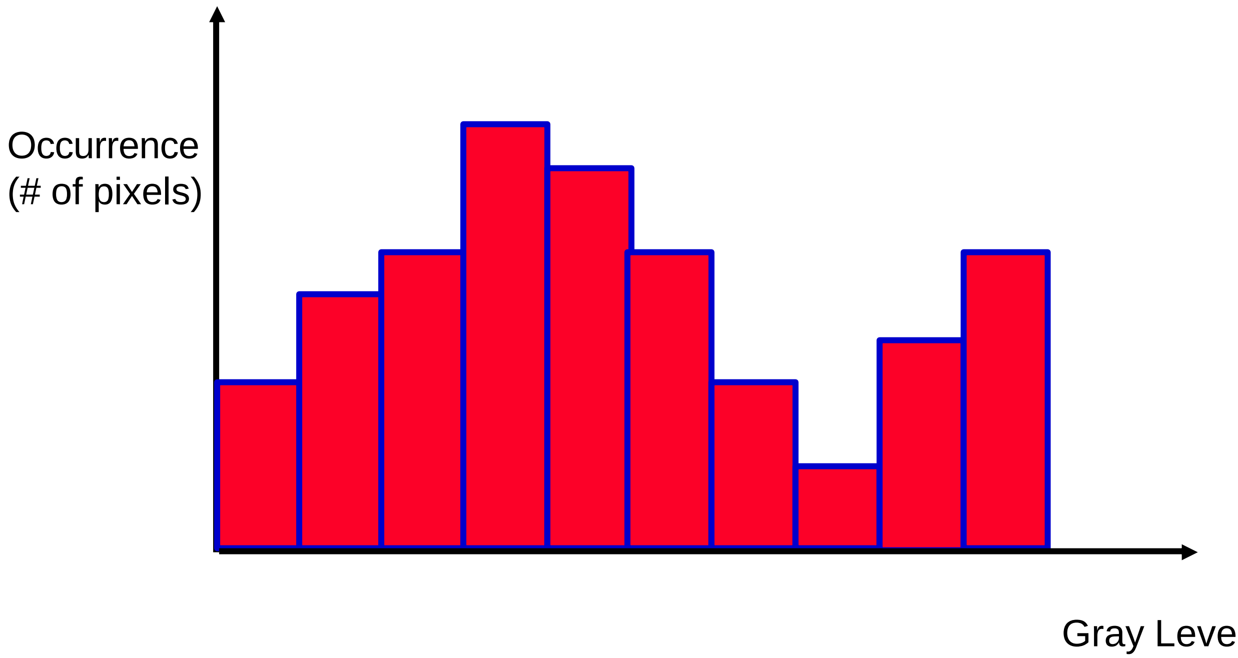
Solution:

$$q_i = \frac{\int_{Z_i}^{Z_{i+1}} ZP(Z) dz}{\int_{Z_i}^{Z_{i+1}} P(Z) dz}$$

(weighted average in the range [Z_i ... Z_{i+1}])

$$Z_i = (q_{i-1} + q_i)/2$$

Discrete Histogram



H(k) = #pixels with gray-level k

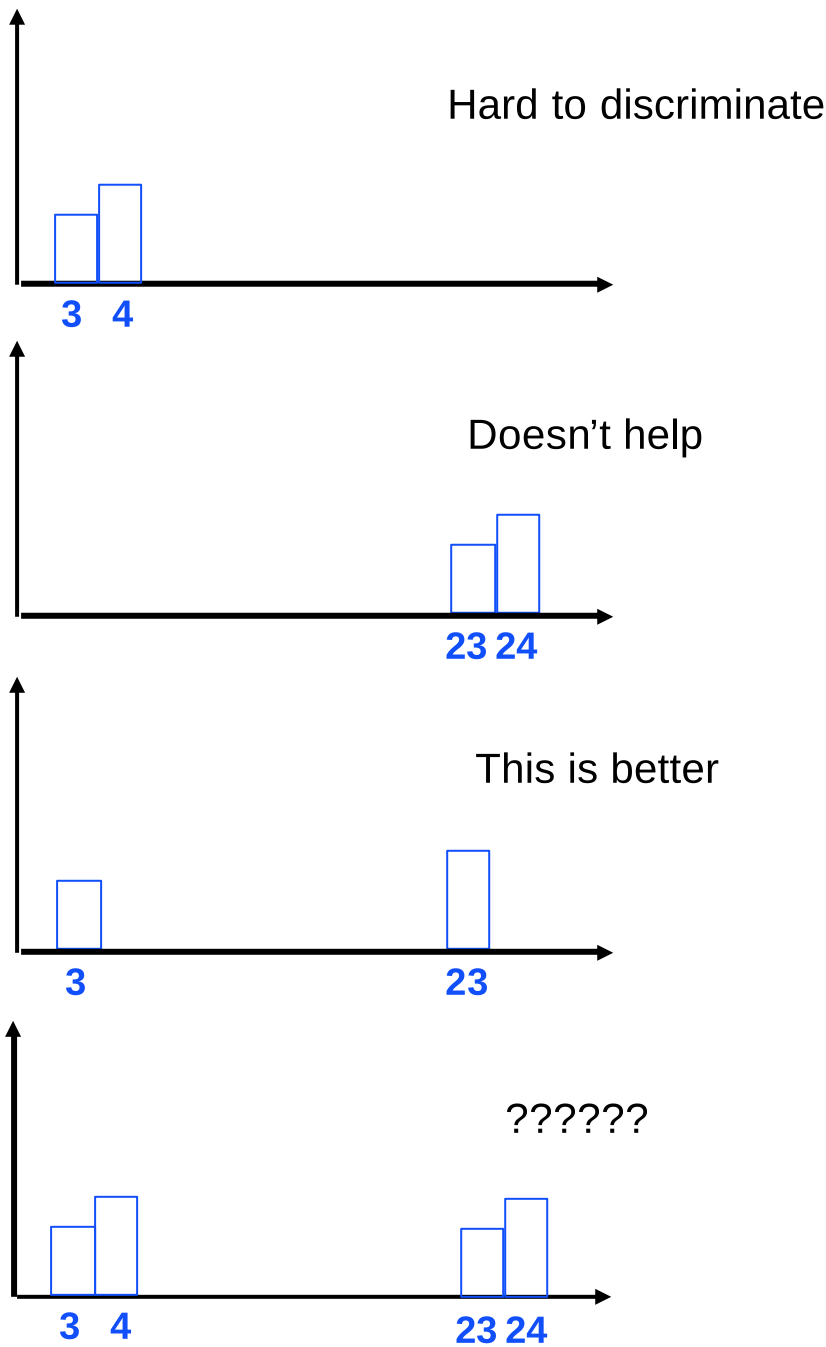
Normalized histogram:

$H_{norm}(k)=H(k)/N$

where N is the total number of pixels in the image.

Gray Level Separation:

Visual discrimination between objects depends on the their gray-level separation. Can we improve discrimination AFTER image has been quantized?



Histogram Equalization

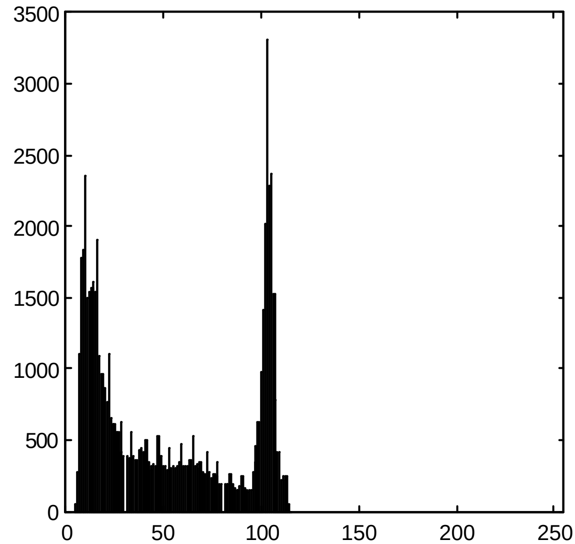
- For a better visual discrimination we would like to re-assign gray-levels with maximal uniformity.
- Define a gray-level transformation

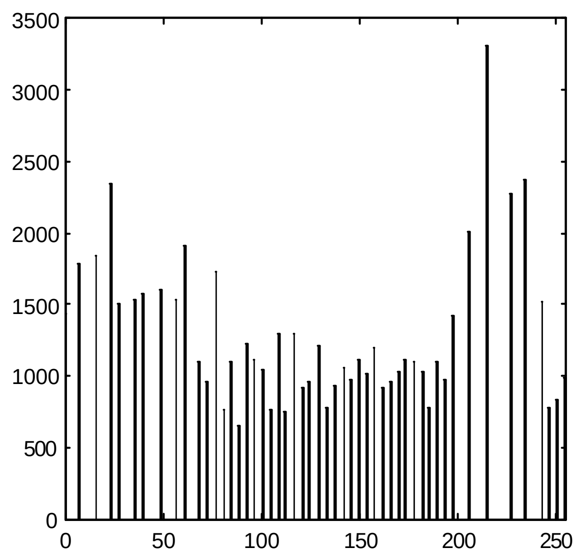
$$\hat{g} = T(g)$$

such that:

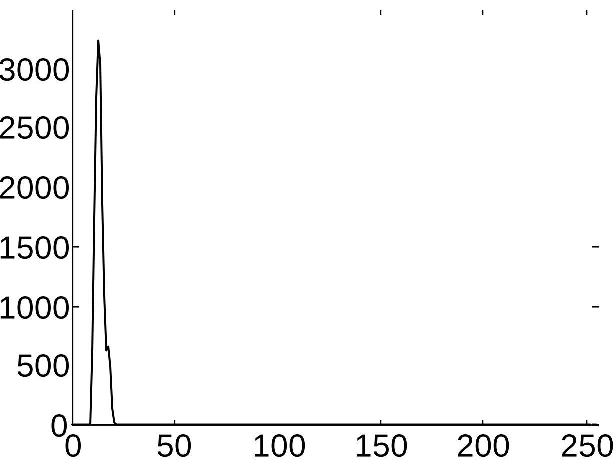
- The histogram according to $\hat{g}$ is as flat as possible.
- The order of Gray-levels is maintained.
- The histogram bars are not fragmented.
- For example:

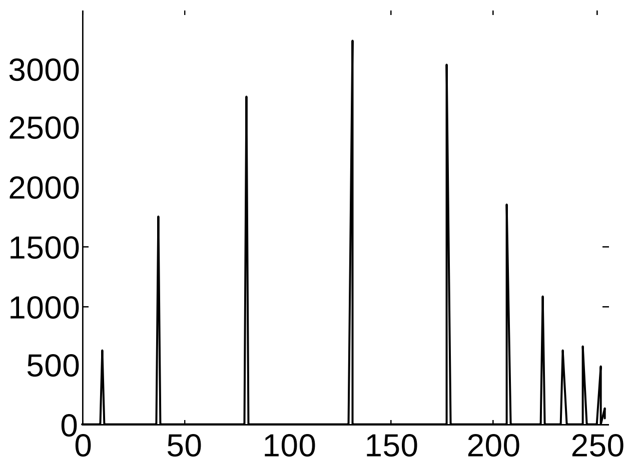
$$T(g) = \frac{H(0) + H(1) + ... + H(g)}{N} \cdot 255$$


3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250


3500
3000
2500
2000
1500
1000
500
0
0
50
100
150
200
250

Histogram Equalization - Example





Original

Equalized

Example Questions

1. Given an image $I(x,y) \in [0,1]$.

   How can we obtain the maximum contrast image using a linear gray-level transformation:

$$I_{NEW}(x,y)=a\ I(x,y)+b$$

   – Find a and b.

   – Given the histogram of I, describe, the histogram of I_{NEW}.

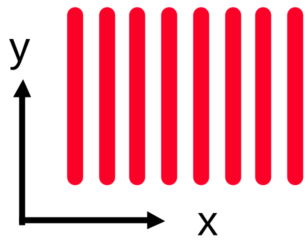
   – Hints:

     • Use the values MAX(I) and MIN(I) which are the maximal and minimal gray-level values in the image.

     • The new gray-level values should be kept in the range [0,1].

2. Given the histogram H(I) what is the average of the image I ?

3. In the following cyclic pattern the frequency in the X direction is 20 cycles/length.



– What is the wavelength of this pattern in the X direction?
– What is the frequency and wavelength of this pattern in the Y direction?
– What is the frequency and wavelength (for X and Y) of this pattern after rotating it by 30 degrees clockwise?