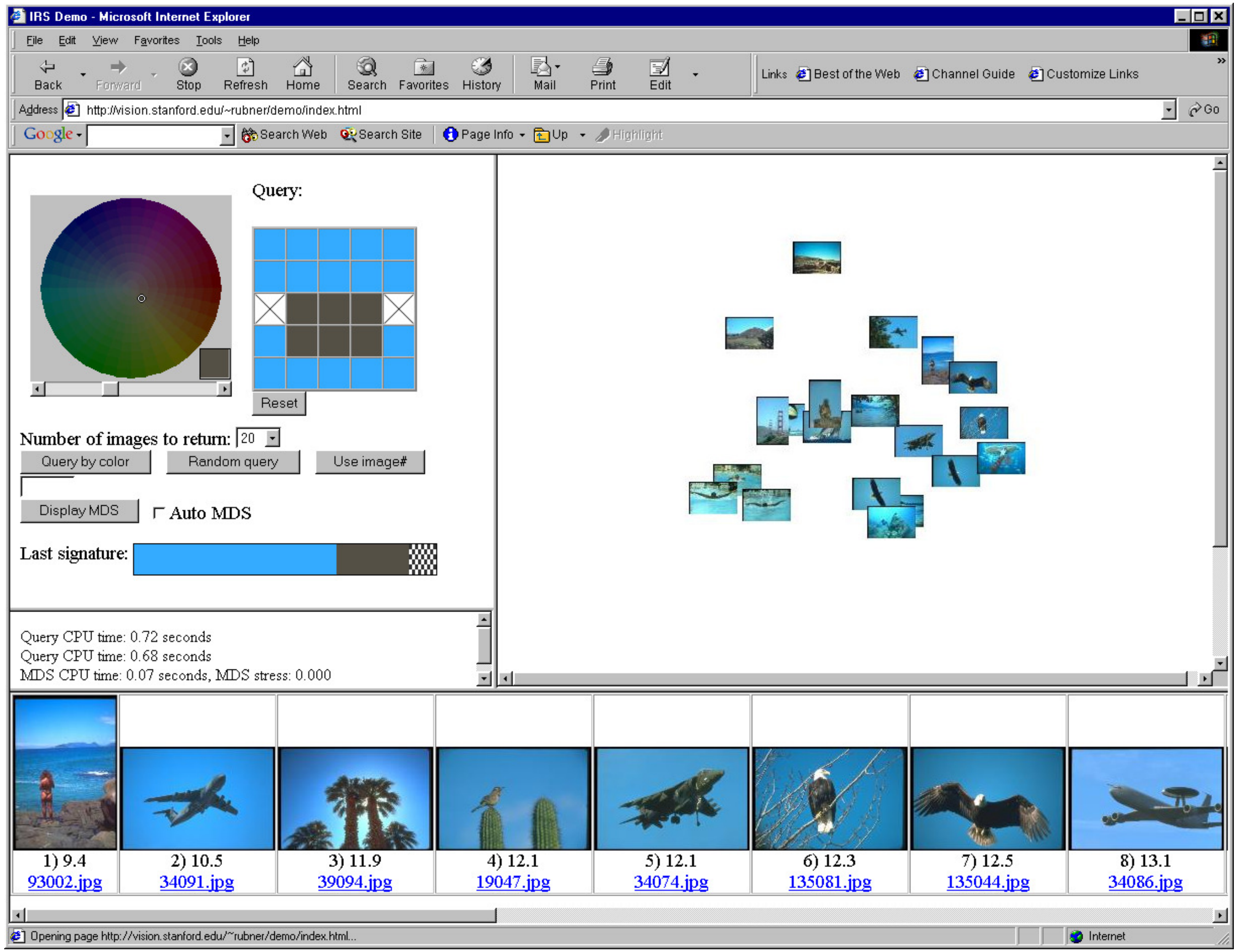
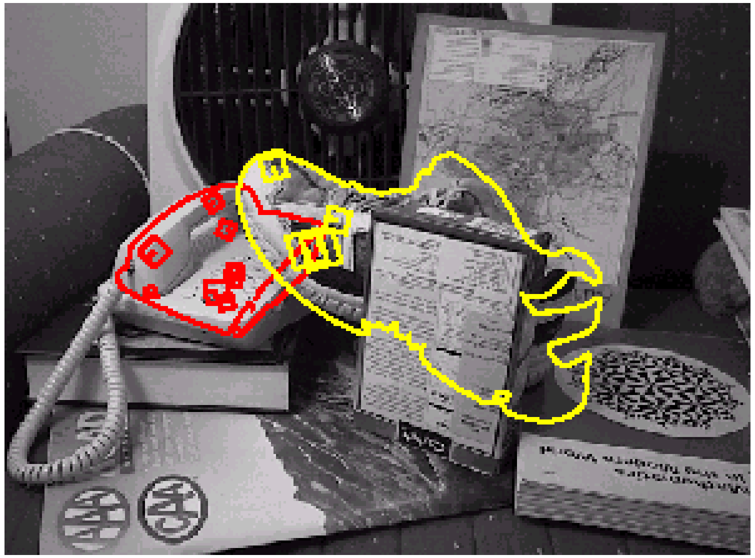
Image Matching

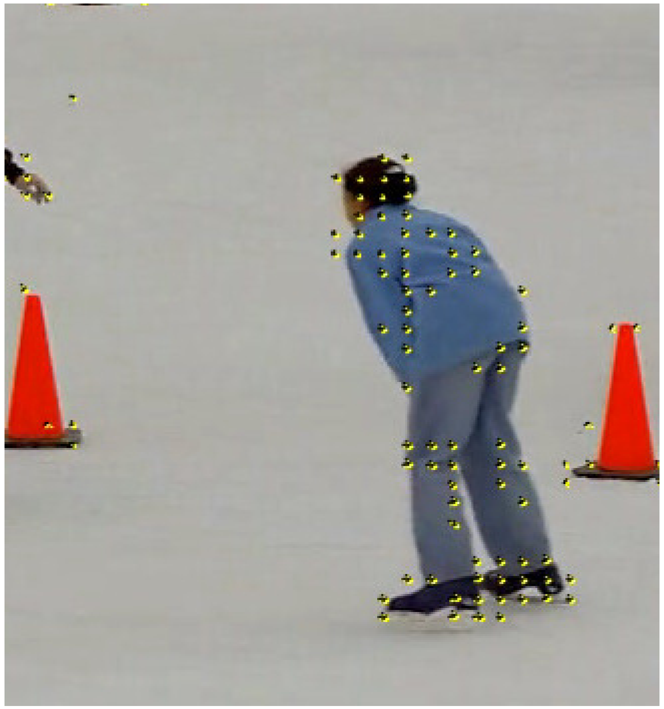
Image Retrieval

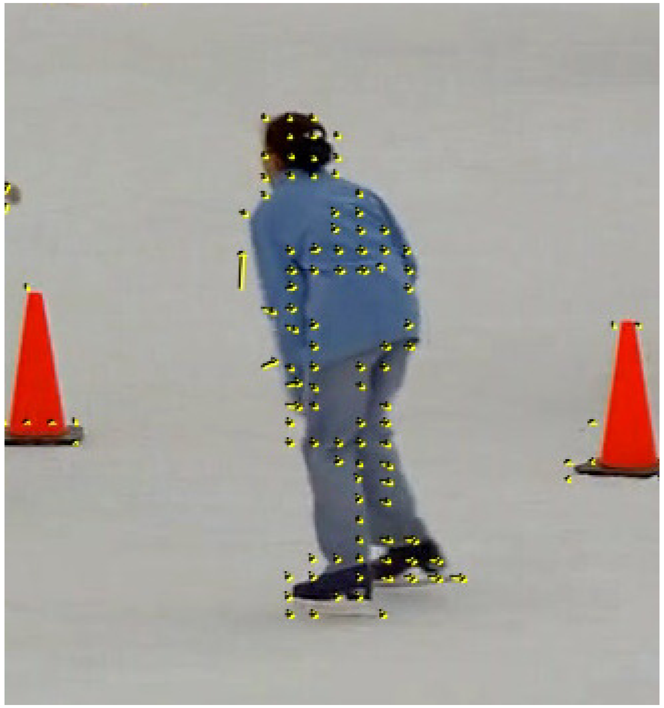


Object Recognition



Motion Estimation and Optical Flow Tracking

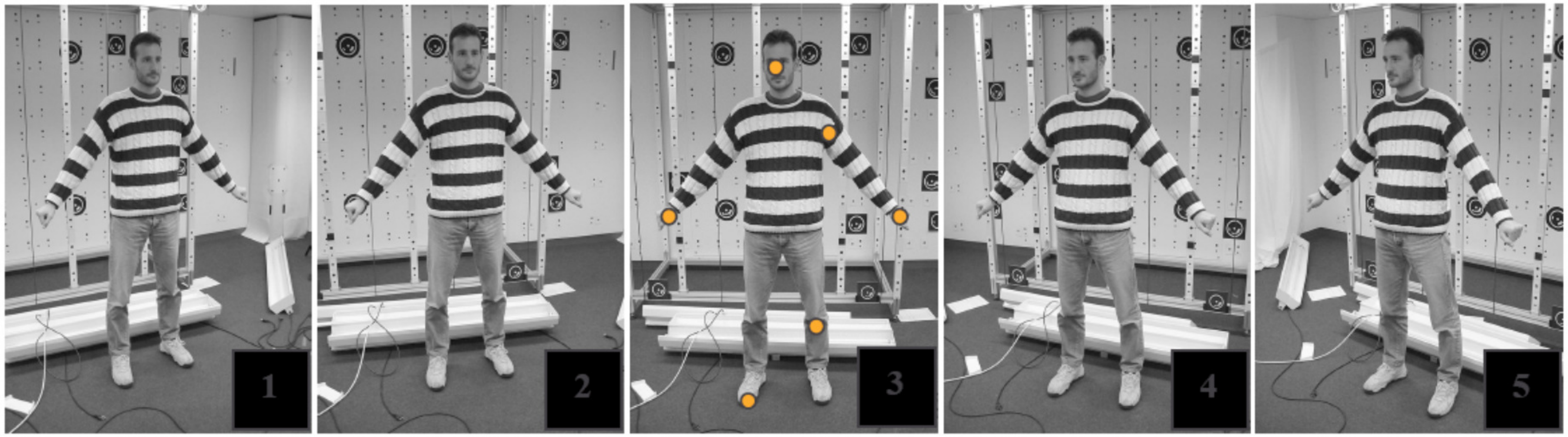




Example: Mosiacing (Panorama)

M. Brown and D. G. Lowe. Recognising Panoramas. ICCV 2003

Example – 3D Reconstruction



Source: http://www.photogrammetry.ethz.ch/general/persons/fabio/fabio_spie0102.pdf

Image Matching

Three approaches:

- **Shape Matching**
  - Assume shape has been extracted
- **Direct (appearance-based) registration**
  - Search for alignment where most pixels agree
- **Feature-based registration**
  - Find a few matching features in both images
  - compute alignment

Direct Method (brute force)

The simplest approach is a brute force search

- Need to define image distance function:
  SSD, Normalized Correlation, Mutual Information, etc.
- Search over all parameters within a reasonable range:

  e.g. for translation:

for Δx=x0:step:x1,
  for Δy=y0:step:y1,
    calculate *Dist*(image1(x,y),image2(x+Δx,y+Δy))
  end;
end;

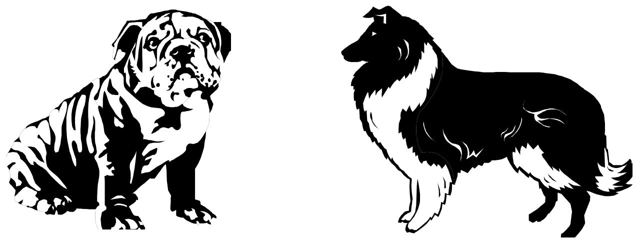
Shape Representation

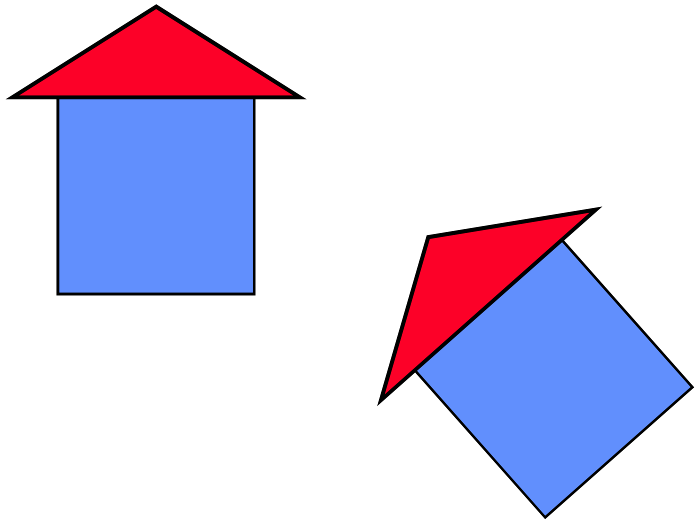
- Region Based Representation
- Area / Circumference / Width
- Euler Number

- Moments
- Quad Trees

- Edge Based Representation
- Chain Code
- Fourier Descriptor

- Interior Based Representation
- MAT / Skeleton
- Hierarchical Representations

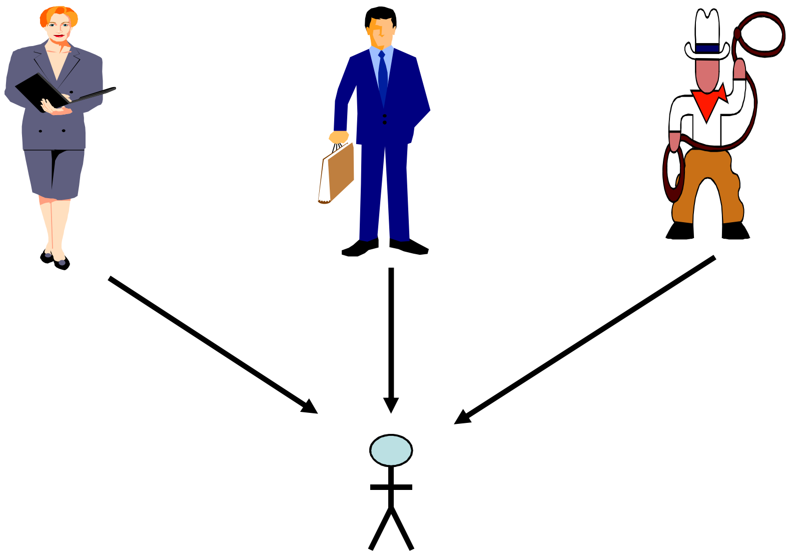




Shape Representation

Shape representation must be GOOD:

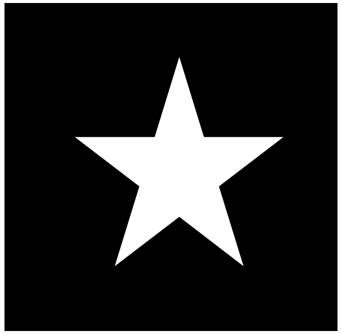
- Different shapes ⇔ Different Codes
- Location / Rotation /Scale Invariant
- Convenient
- Stable
- Generative



Moments

$$I(x,y) = \begin{cases} 1 & \textbf{If pixel (x,y) is IN object} \\ 0 & \textbf{otherwise} \end{cases}$$

ij –Moment: $$M_{ij} = \sum_x \sum_y x^i y^j I(x,y)$$



Area: $$M_{00} = \sum_x \sum_y I(x,y)$$

Average x-coordinate:
Average y-coordinate:

$$\bar{x} = \frac{M_{10}}{M_{00}} \qquad \bar{y} = \frac{M_{01}}{M_{00}}$$

Center of Mass: $$(\bar{x}, \bar{y}) = \left(\frac{M_{10}}{M_{00}}, \frac{M_{01}}{M_{00}} \right)$$

Moments

Central Moment: $$\mu_{ij} = \sum_x \sum_y (x - \bar{x})^i (y - \bar{y})^j I(x, y)$$

Moment expressions that are invariant to translation, rotation and/or scale:

1. For first-order moments, $\mu_{0,1} = \mu_{1,0} = 0$, (always invariant).
2. For second-order moments, $(p + q = 2)$, the invariants are

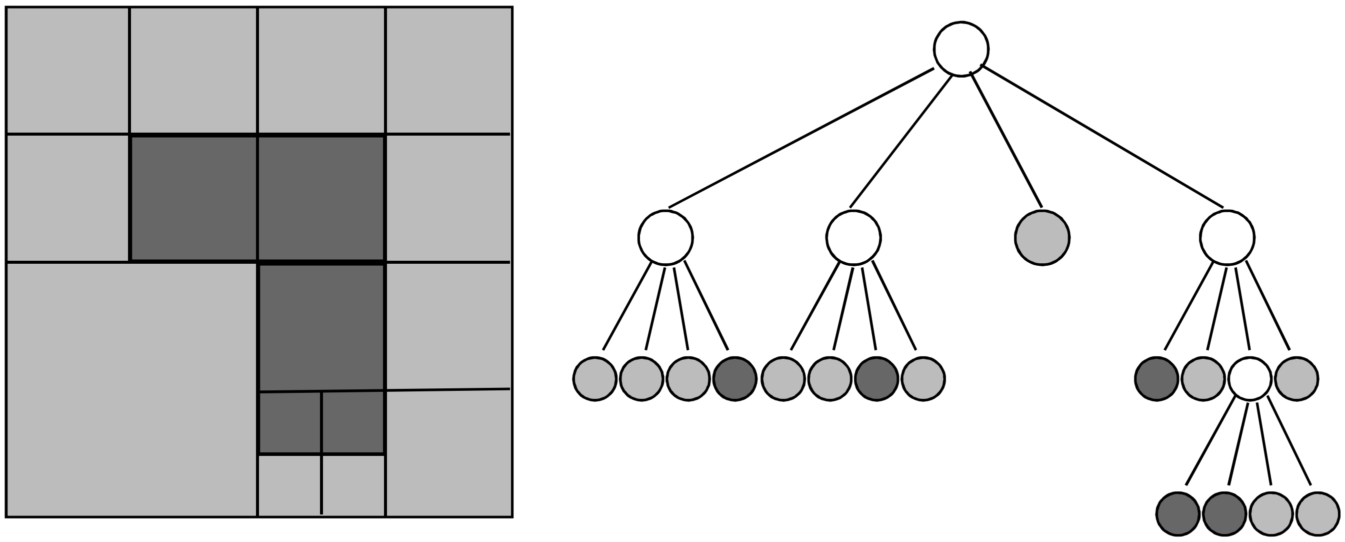
$$\phi_1 = \mu_{2,0} + \mu_{0,2}$$
$$\phi_2 = (\mu_{2,0} - \mu_{0,2})^2 + 4\mu_{1,1}^2 \qquad (9.80)$$

3. For third-order moments $(p + q = 3)$, the invariants are

$$\phi_3 = (\mu_{3,0} - 3\mu_{1,2})^2 + (\mu_{0,3} - 3\mu_{2,1})^2$$
$$\phi_4 = (\mu_{3,0} + \mu_{1,2})^2 + (\mu_{0,3} + \mu_{2,1})^2$$

wide domain, not unique, not unambiguous, not generative, not stable, invariant to translation, rotation. Very convenient.

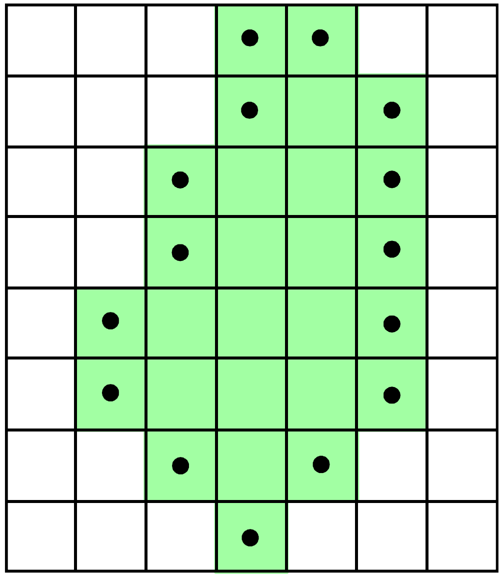
Quad Tree Representation

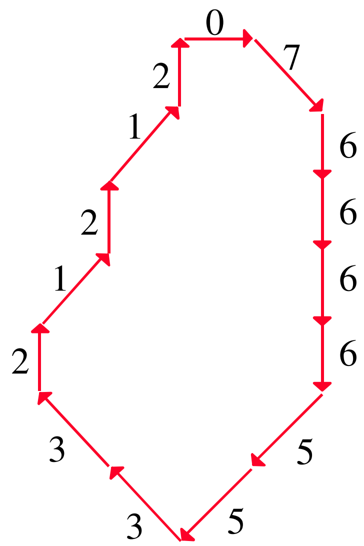


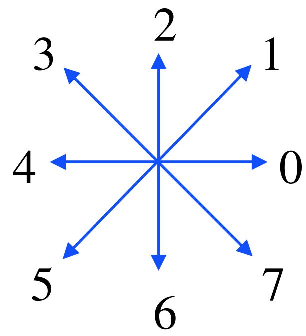
wide domain,
unique, unambiguous, generative – up to error tolerance
partially stable
Not invariant to translation, rotation scale.
Inefficient for comparison

Edge Based Representation

Chain Code



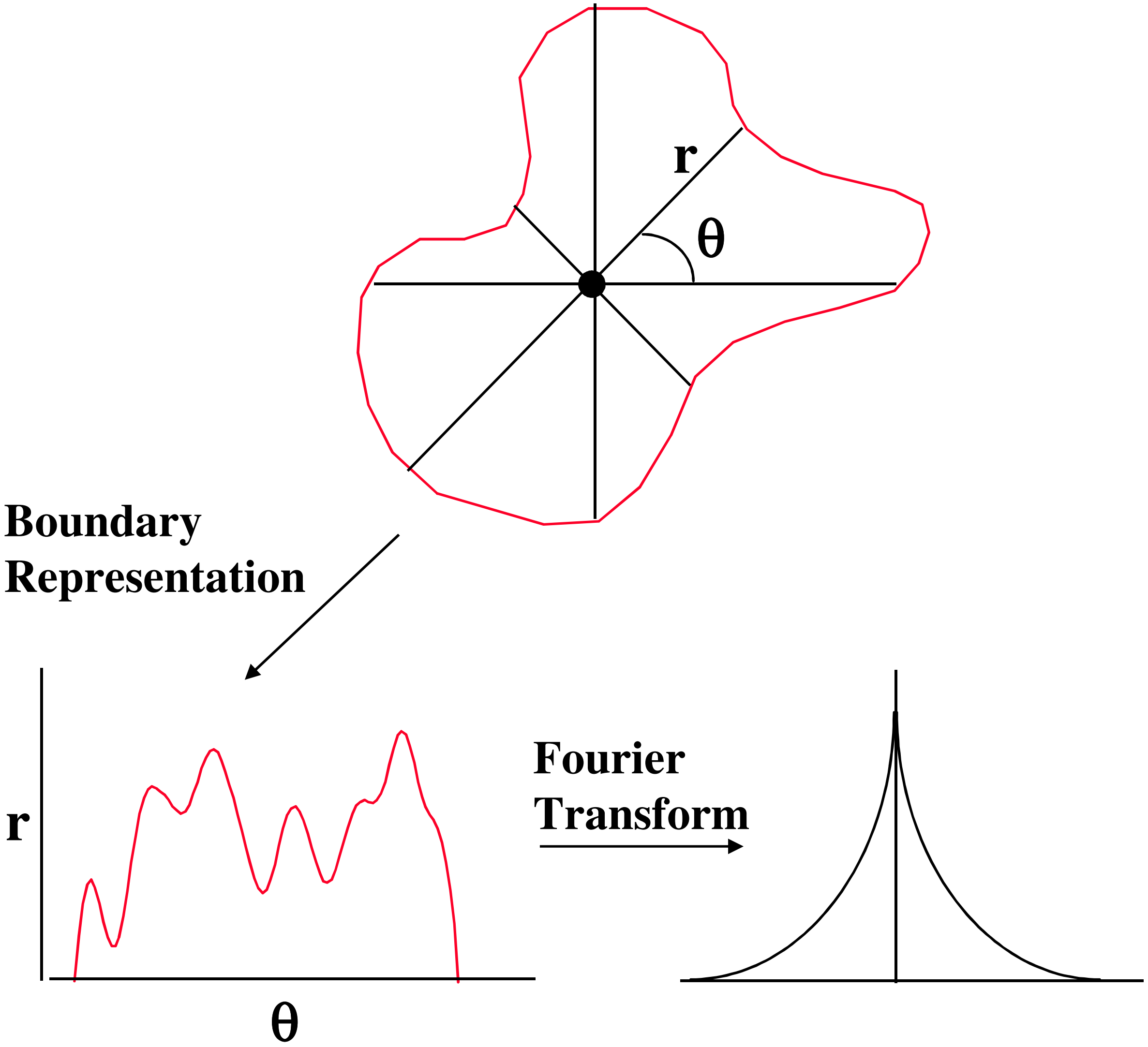


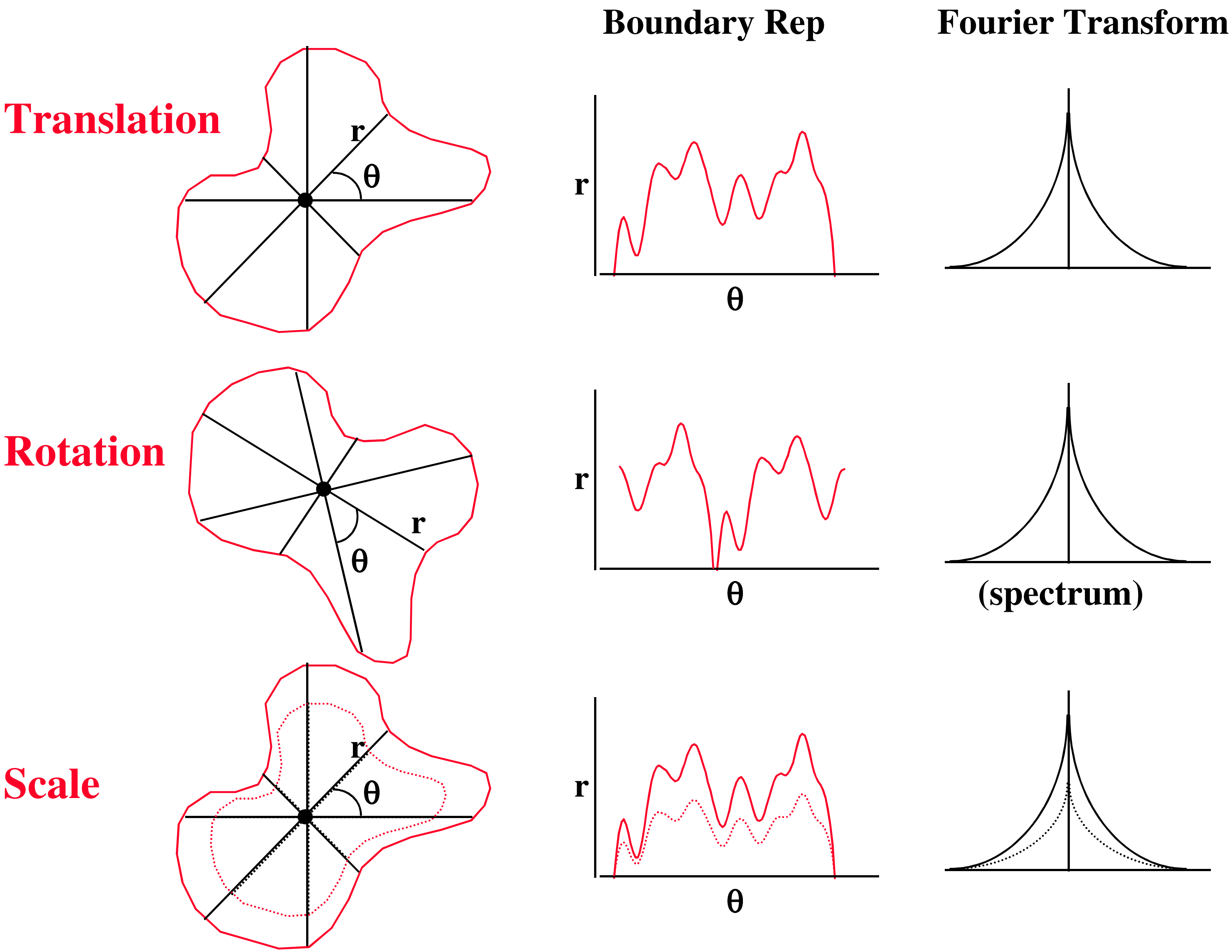


000102011717211

wide domain, Unique, unambiguous, generative - 2D only, Not very stable Invariant to translation. Rotation (x90 deg)

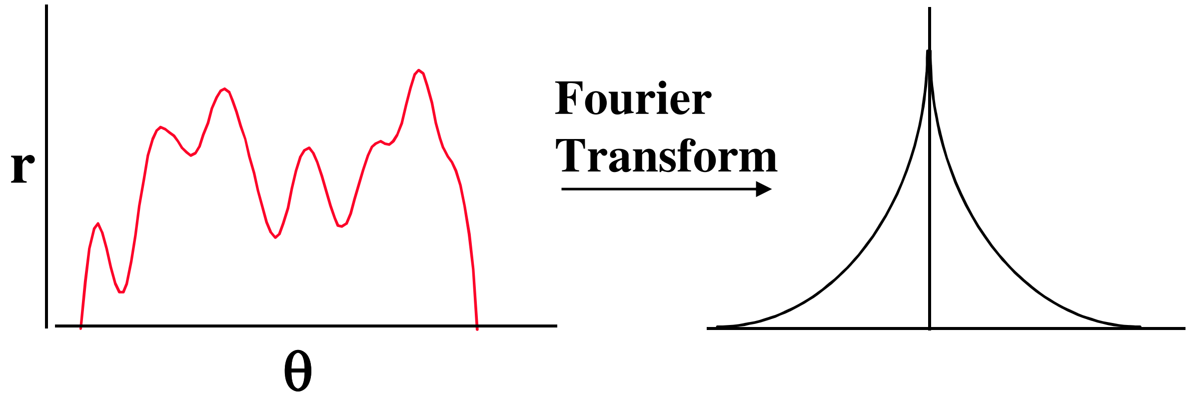
Fourier Descriptors





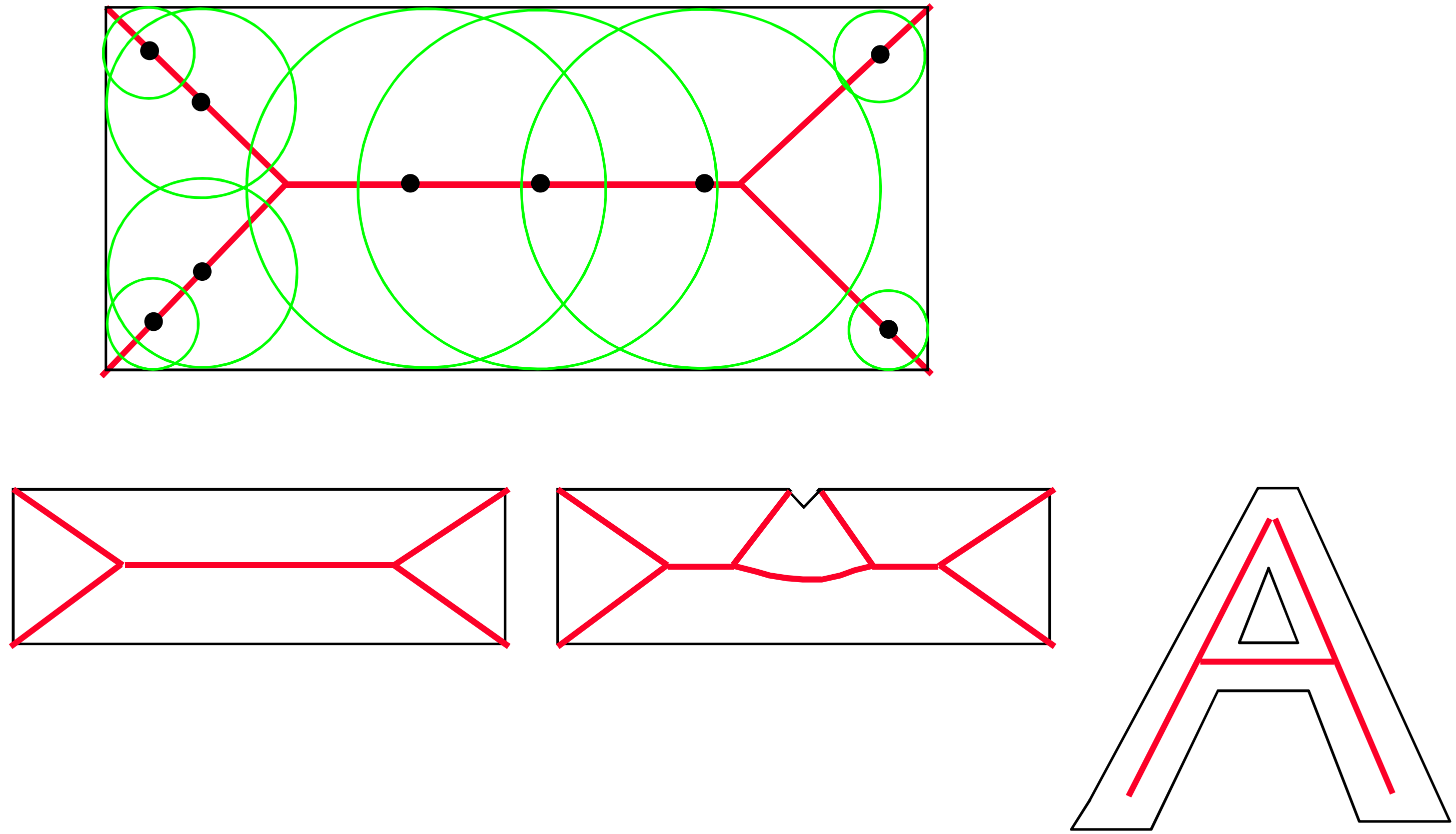
Boundary Rep
Fourier Transform
Translation
r
θ
r
θ
Rotation
r
θ
r
θ
(spectrum)
Scale
r
θ
r
θ

Fourier Descriptors



wide domain, Unique, unambiguous, generative, Stable (depends on tolerance), Invariant to translation. Rotation, Scale.

Interior Based representation – MAT, Skeleton



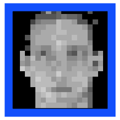
wide domain, unique, unambiguous, generative

not stable - small changes affect dramatically

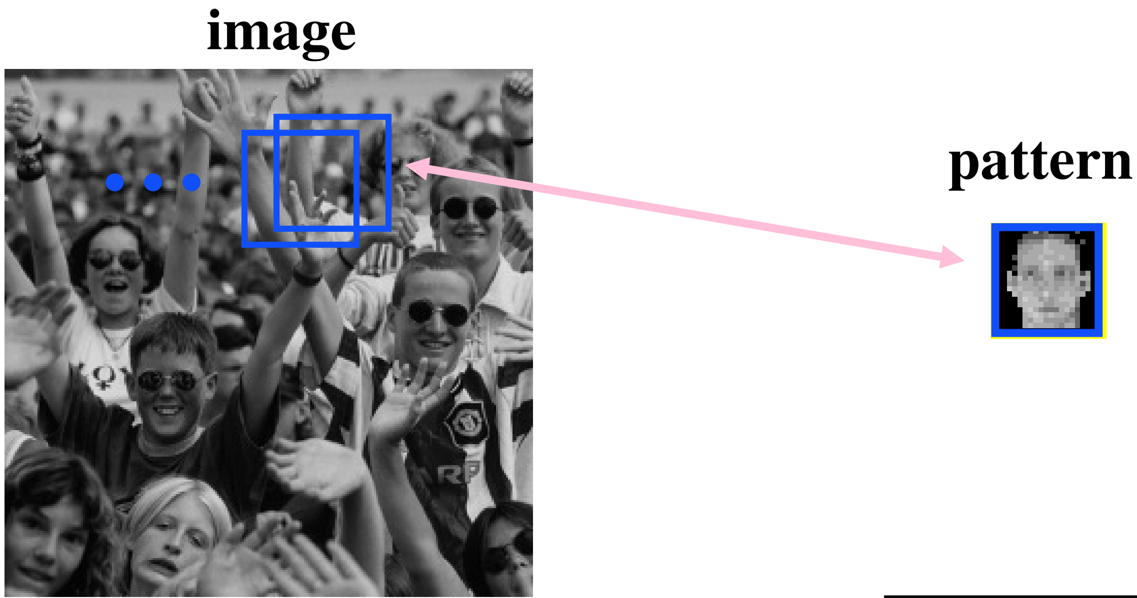
Pattern Matching – Direct approach (Appearance based)

image

pattern

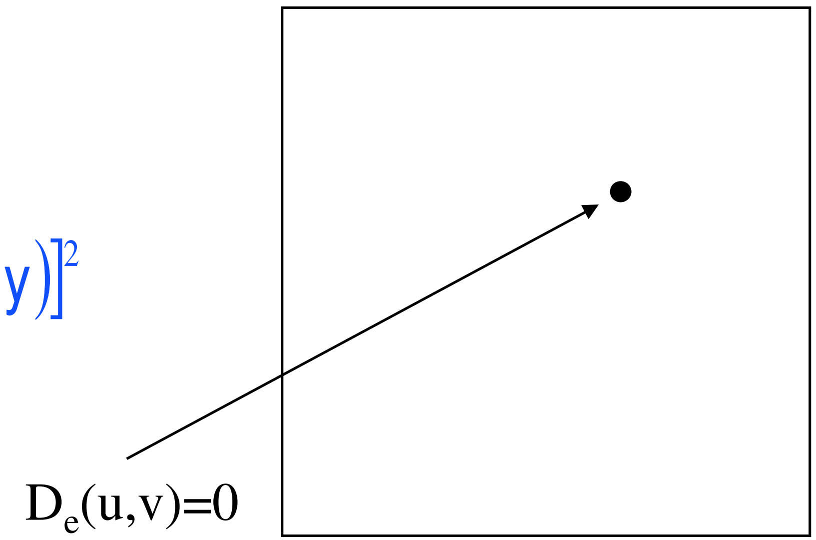


Finding a pattern in an Image



Look for minimum of:

$$d_e(u,v) = \sum_{x,y \in N} \left[I(u+x, v+y) - P(x,y) \right]^2$$



Finding a pattern in an Image

$$d_e(u,v) = \sum_{x,y \in N} \left[I(u+x, v+y) - P(x,y) \right]^2$$

$$= \sum_{x,y \in N} I(u+x, v+y)^2 + P(x,y)^2 - 2I(u+x, v+y)P(x,y)$$

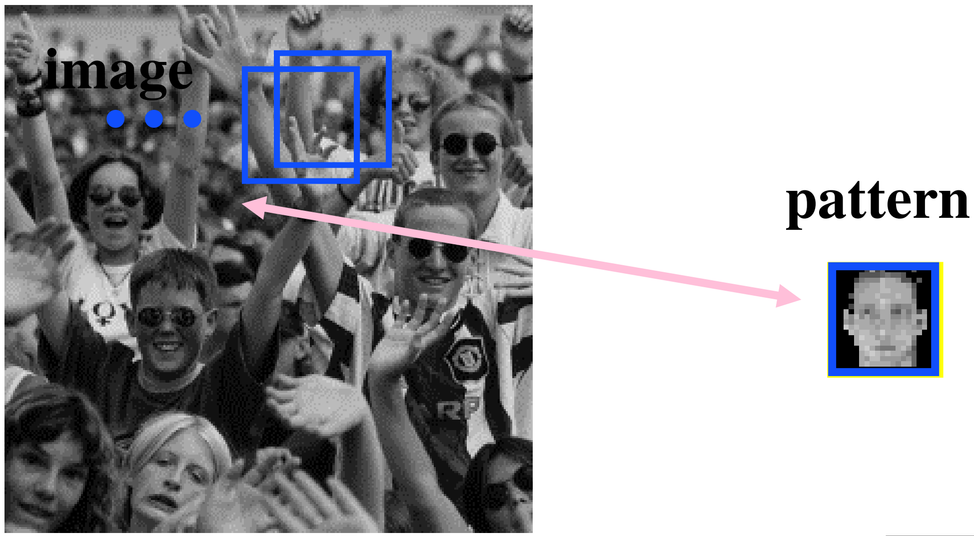
$$= \sum_{x,y \in N} I(u+x, v+y)^2 + \sum_{x,y \in N} P(x,y)^2 - 2\sum_{x,y \in N} I(u+x, v+y)P(x,y)$$

Sum of squares of the window

Sum of squares of the pattern CONSTANT

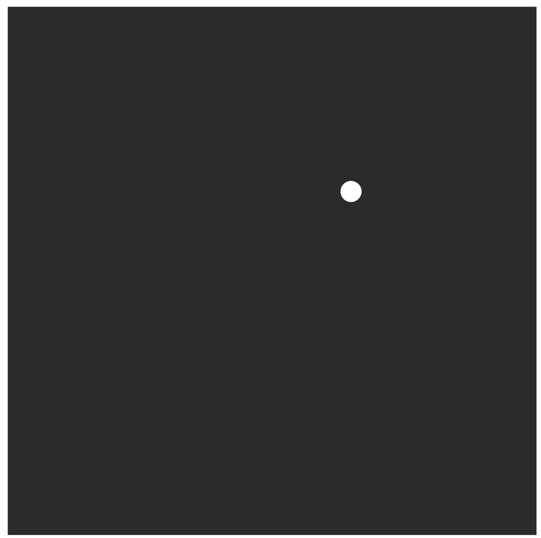
Correlation

Finding a pattern in an Image - Correlation



Look for maximum of:

$$\sum_{x,y \in N} \left[I(u+x, v+y) P(x,y) \right]$$

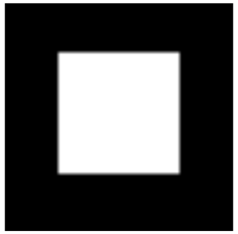


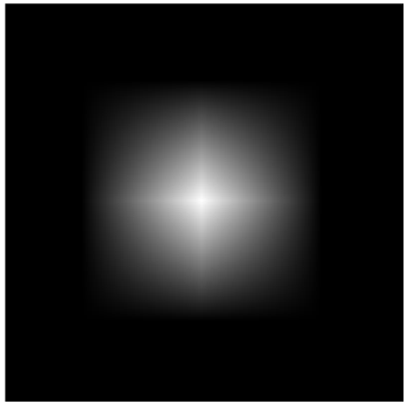
Correlation

I

*

P



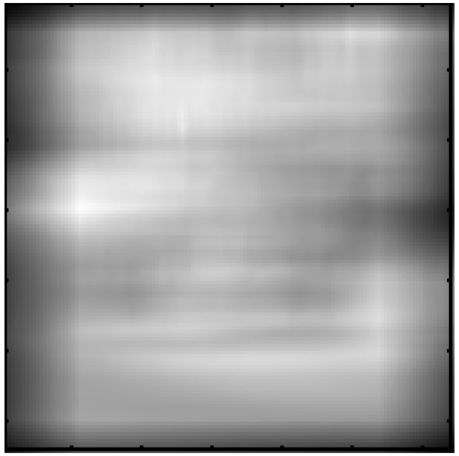


I corr P

Real Image – Correlation Example

image

pattern



Correlation

Correlation value is dependent on the local gray value of the pattern and the image window.

Normalized Correlation

$$\frac{\sum_{x,y\in N}\left[I(u+x,v+y)-\bar{I}_{uv}\right]\left[P(x,y)-\bar{P}\right]}{\left[\sum_{x,y\in N}\left[I(u+x,v+y)-\bar{I}_{uv}\right]^2\sum_{x,y\in N}\left[P(x,y)-\bar{P}\right]^2\right]^{1/2}}$$

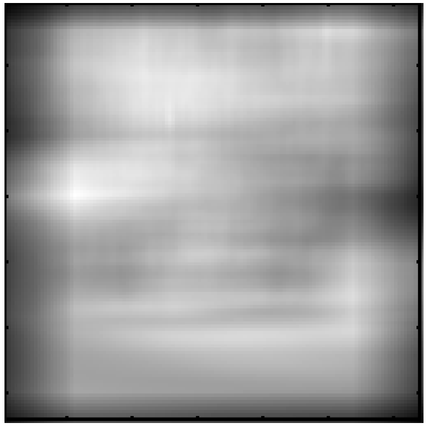
Correlation value is in (-1..1)

Correlation value is **independent** of the local gray value of the pattern and the image window.

Normalized Correlation - Example

image

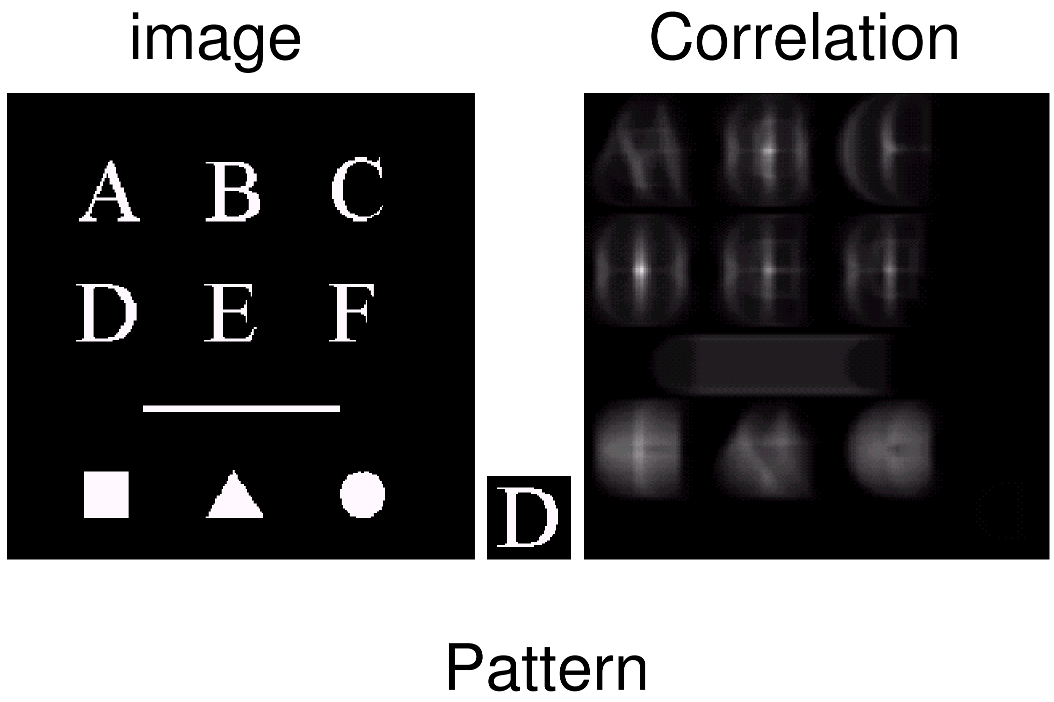
pattern



Correlation

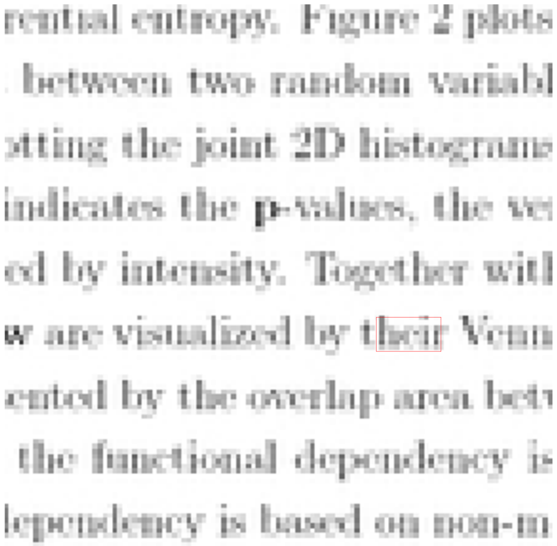
Normalized Correlation

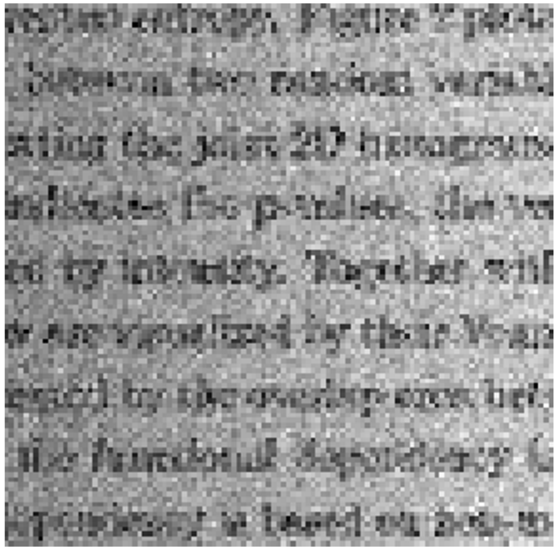
Normalized Correlation - Example



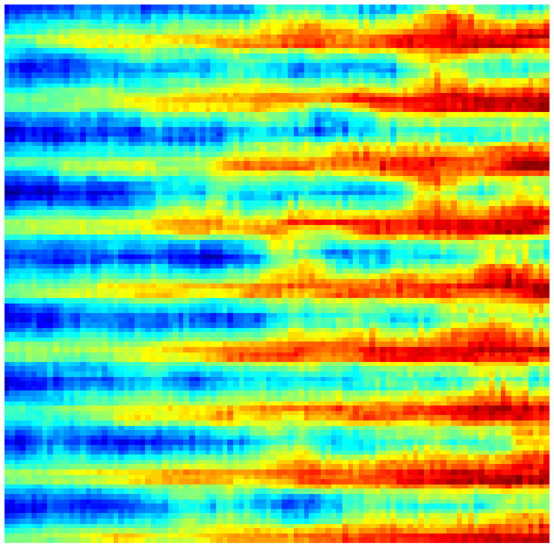
Pattern Matching - Example

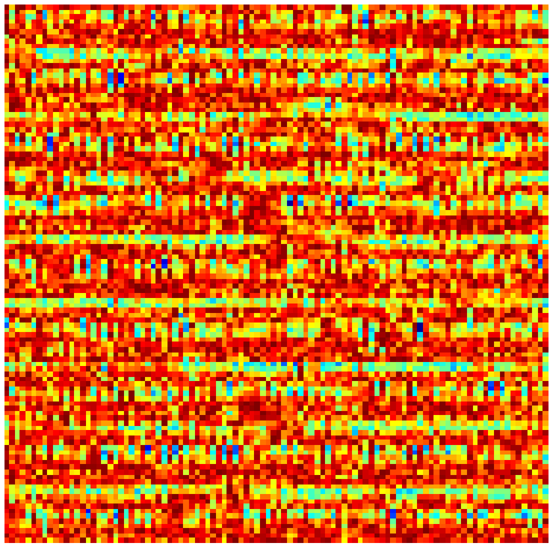
Pattern

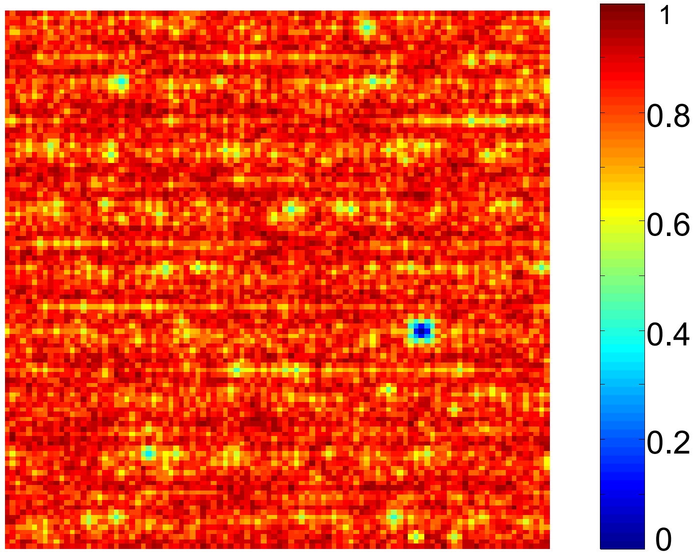




image







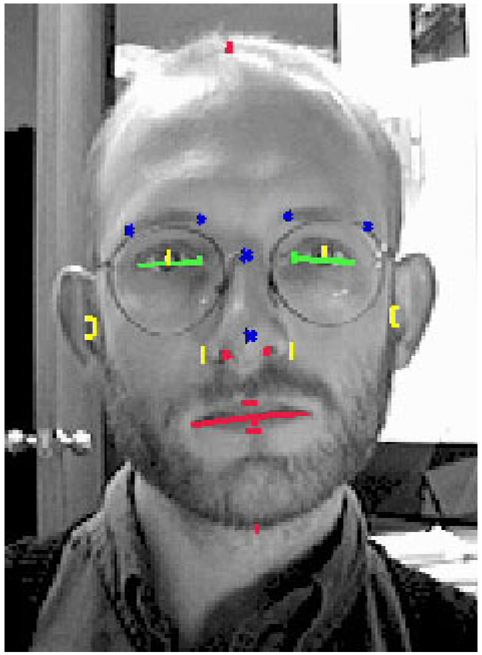
Euclidean

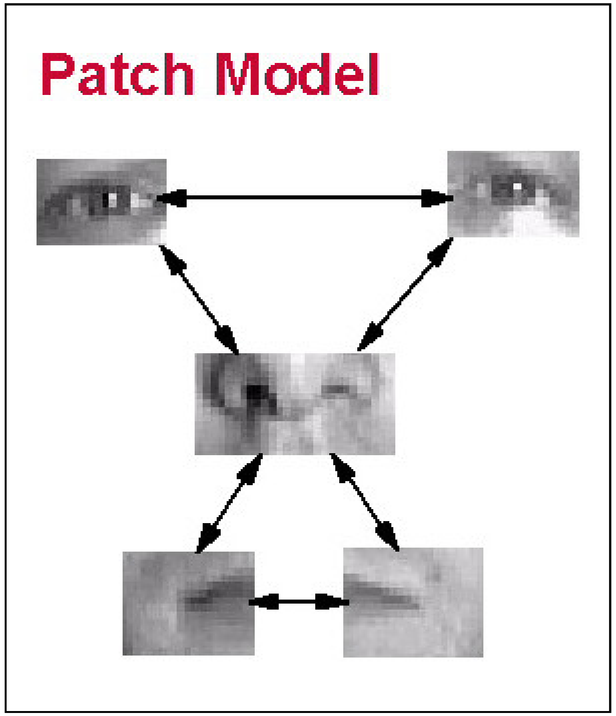
NCC

MTM

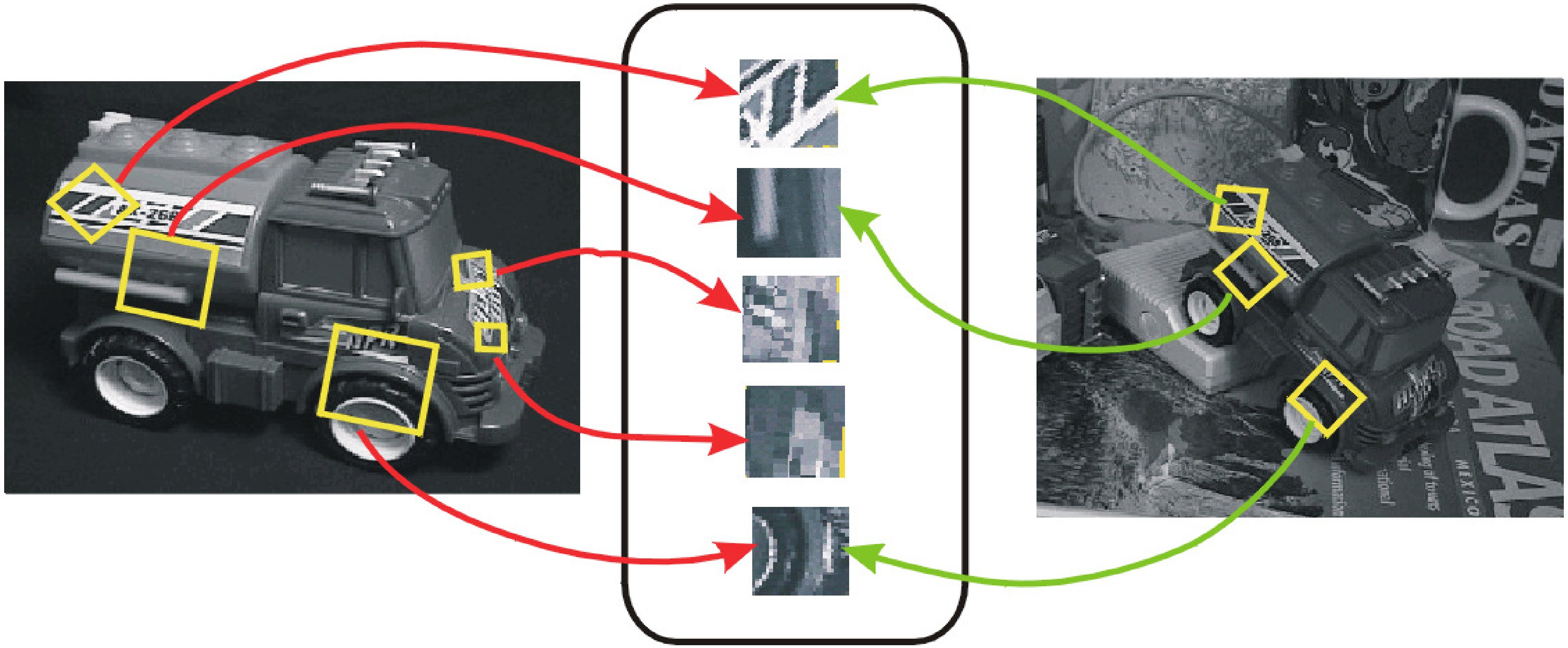
Pairs for Image Matching

Feature Based Object Detection





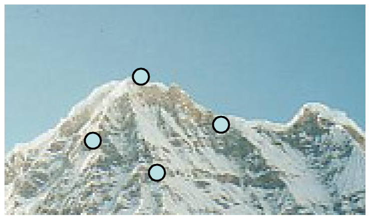
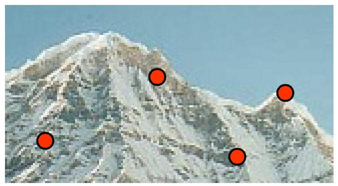
Feature Based Object Detection



Features Descriptors

Features: Issues to be addressed

- What are "good" features to extract?
  - –Distinctive
  - –Invariant to different acquisition conditions
    - –Different view-points, different illuminations, different cameras, etc.
- How can we find corresponding features in both images?

no chance to match!

Invariant Feature Descriptors

- Schmid & Mohr 1997, Lowe 1999, Baumberg 2000, Tuytelaars & Van Gool 2000, Mikolajczyk & Schmid 2001, Brown & Lowe 2002, Matas et. al. 2002, Schaffalitzky & Zisserman 2002

Image Features

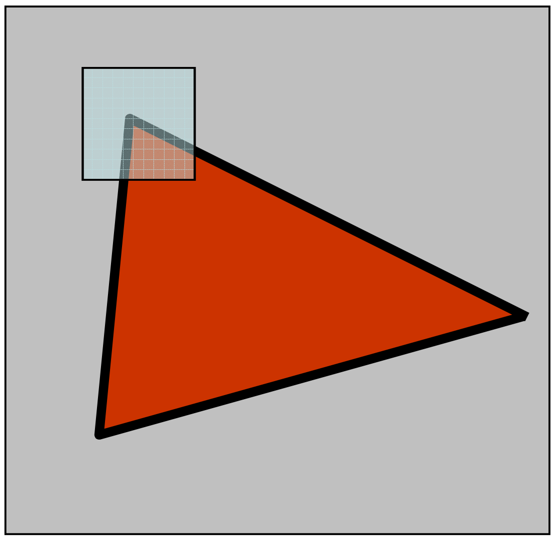
- Feature <u>Detectors</u> - where
- Feature <u>Descriptors</u> - what

- Methods:
  - Harris Corner Detector (multi-scale Harris)
  - SIFT (Scale Invariant Features Transform)

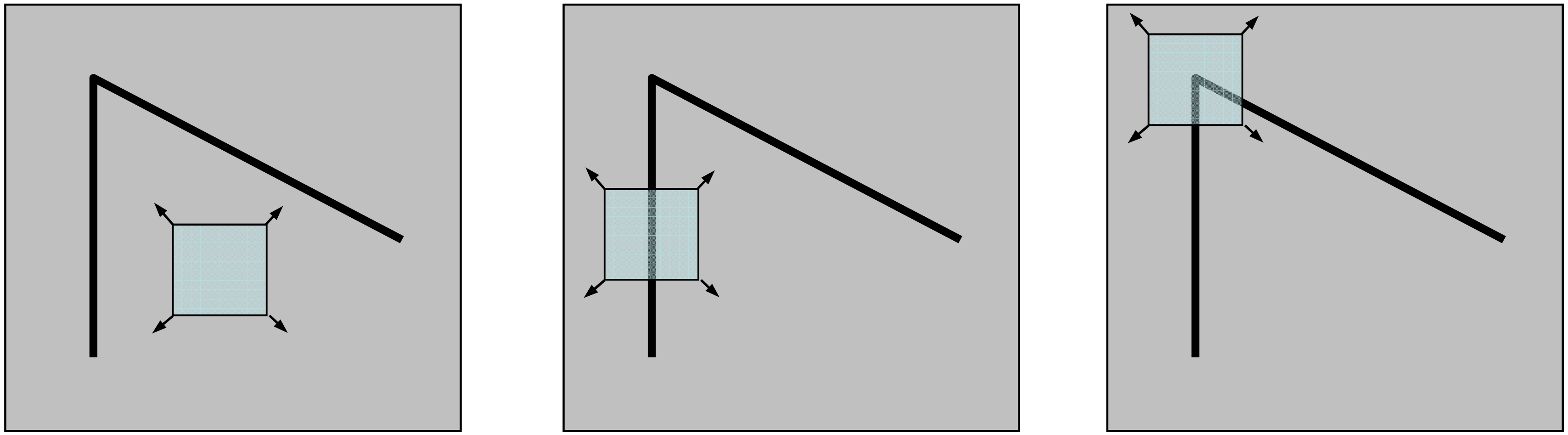
Harris Corner Detector

C.Harris, M.Stephens. “A Combined Corner and Edge Detector”. 1988

- We should easily recognize a corner by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



Harris Detector: Basic Idea



"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

Harris Detector: Mathematics

Corner at position (x,y) ?

Evaluate change of intensity for shift in [*u*,*v*] direction:

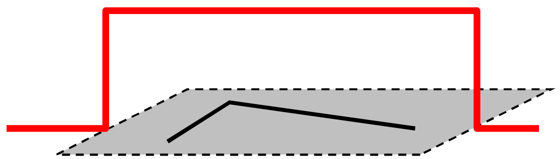
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

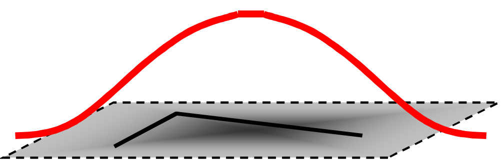
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

For small [u,v]: $I(x+u, y+v) = I(x,y) + uI_x + vI_y$

We have:

$$E(u,v) = \sum_{x,y} w(x,y)\left\| \begin{bmatrix} I_x(x,y) & I_y(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right\|^2 =$$

$$\begin{bmatrix} u & v \end{bmatrix} \sum w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Harris Detector: Mathematics

What is the direction [u,v] of greatest intensity change?

$$\arg\max_{\|(u,v)\|=1} E(u,v) = \mathbf{e}_{\max}$$

Denote by $\mathbf{e}_i$ the i^{th} eigen-vector of M whose eigen-value is λ_i:

$$\mathbf{e}_i^T M \mathbf{e}_i = \lambda_i > 0$$

Conclusions:

$$E(\mathbf{e}_{\max}) = \lambda_{\max}$$

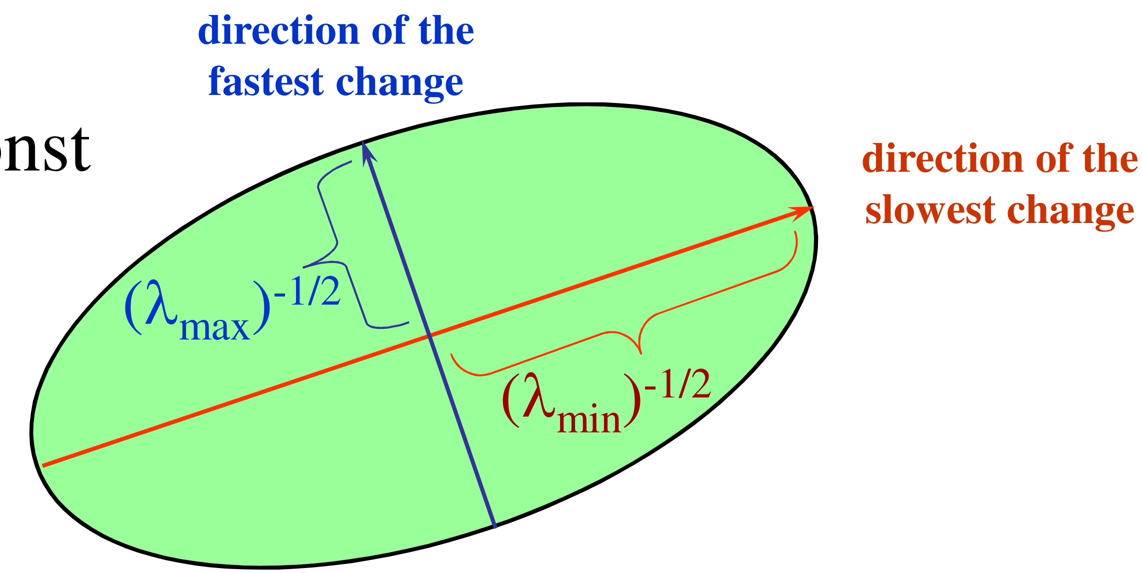
Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

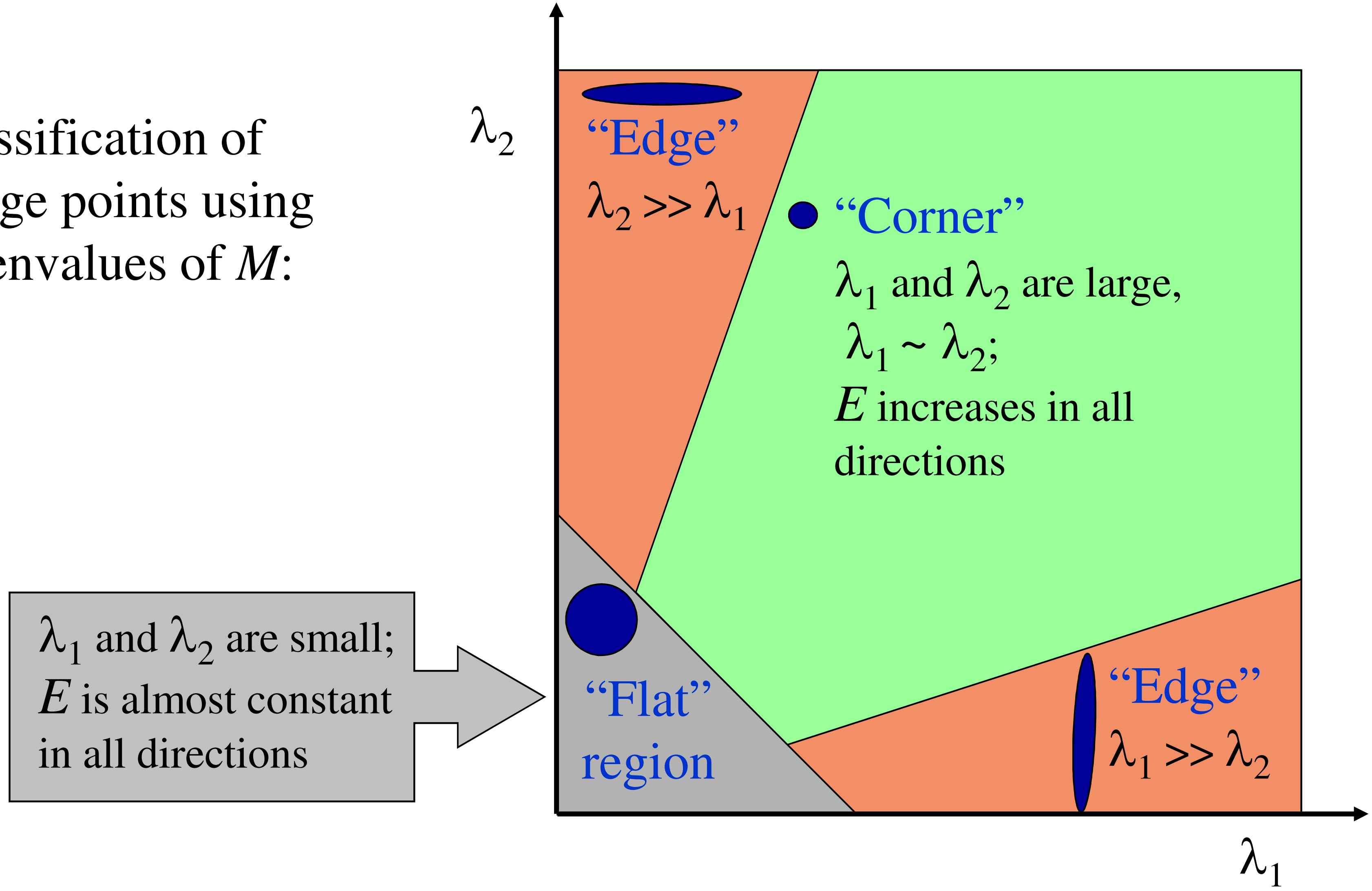
λ_1, λ_2 – eigenvalues of M

Ellipse $E(u,v)$ = const



Harris Detector: Mathematics

Classification of image points using eigenvalues of M:



Harris Detector: Mathematics

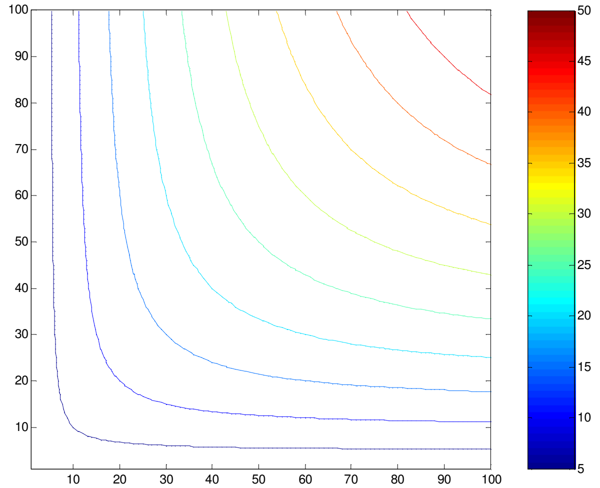
Measure of corner response (without calculating the e.v.):

$$R = \frac{\det M}{\operatorname{Trace} M}$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

R is associated with the smallest eigen-vector (why?)

R v.s. λ_1, λ_2



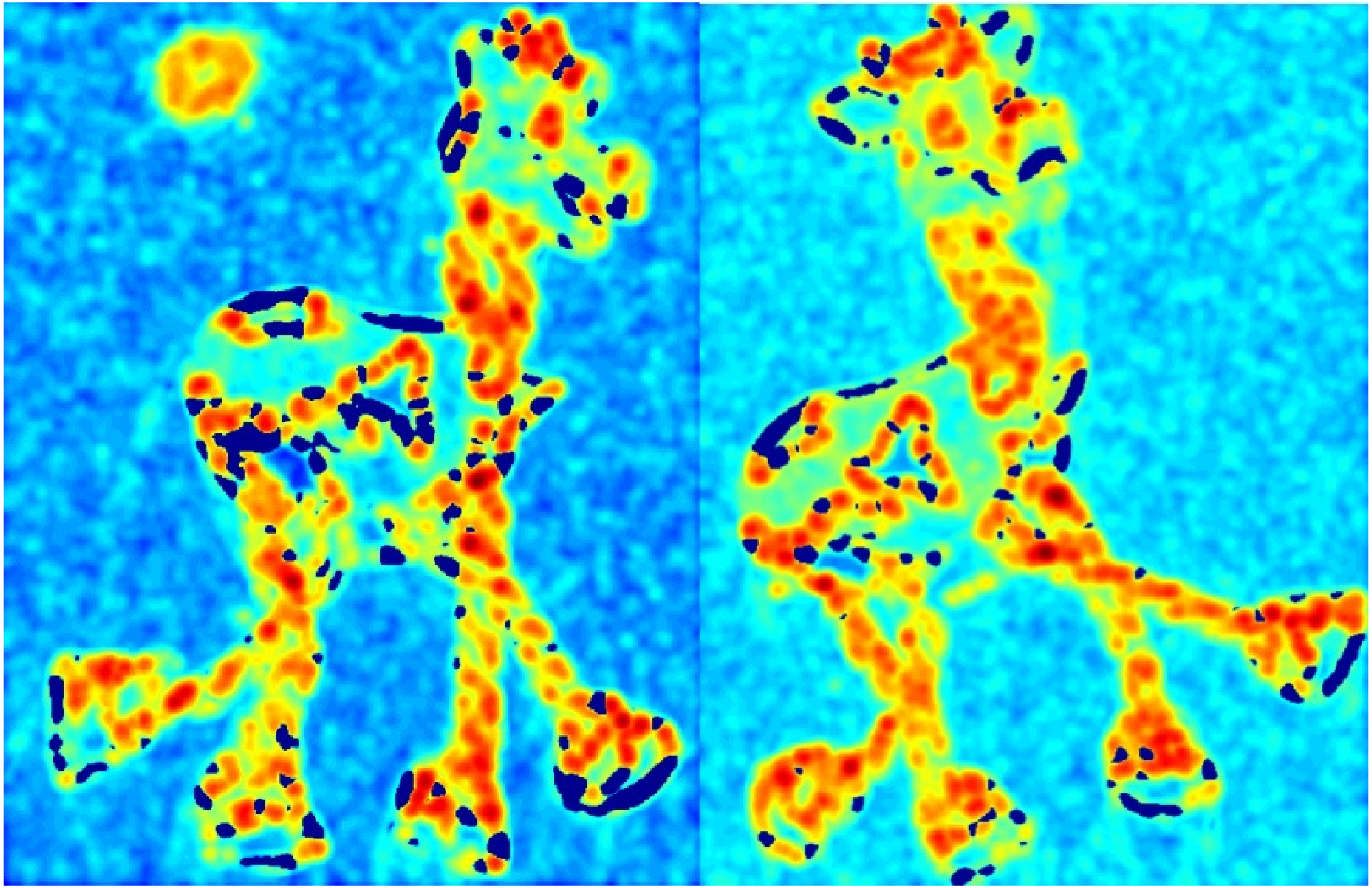
Harris Corner Detector

- The Algorithm:
  - Find points with large corner response function R (R > threshold)
  - Take the points of local maxima of R

Harris Detector: Workflow

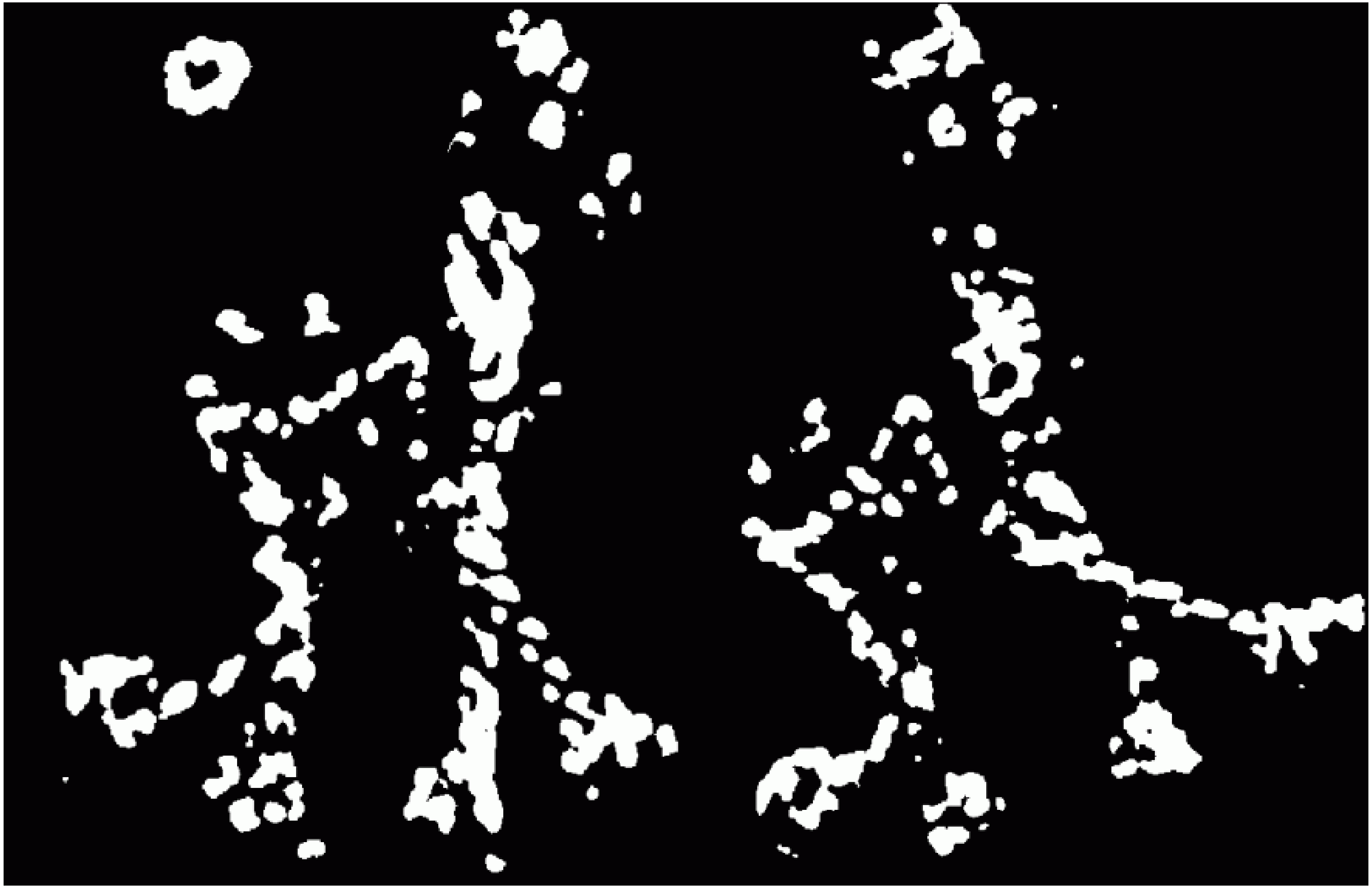
Harris Detector: Workflow

Compute corner response R



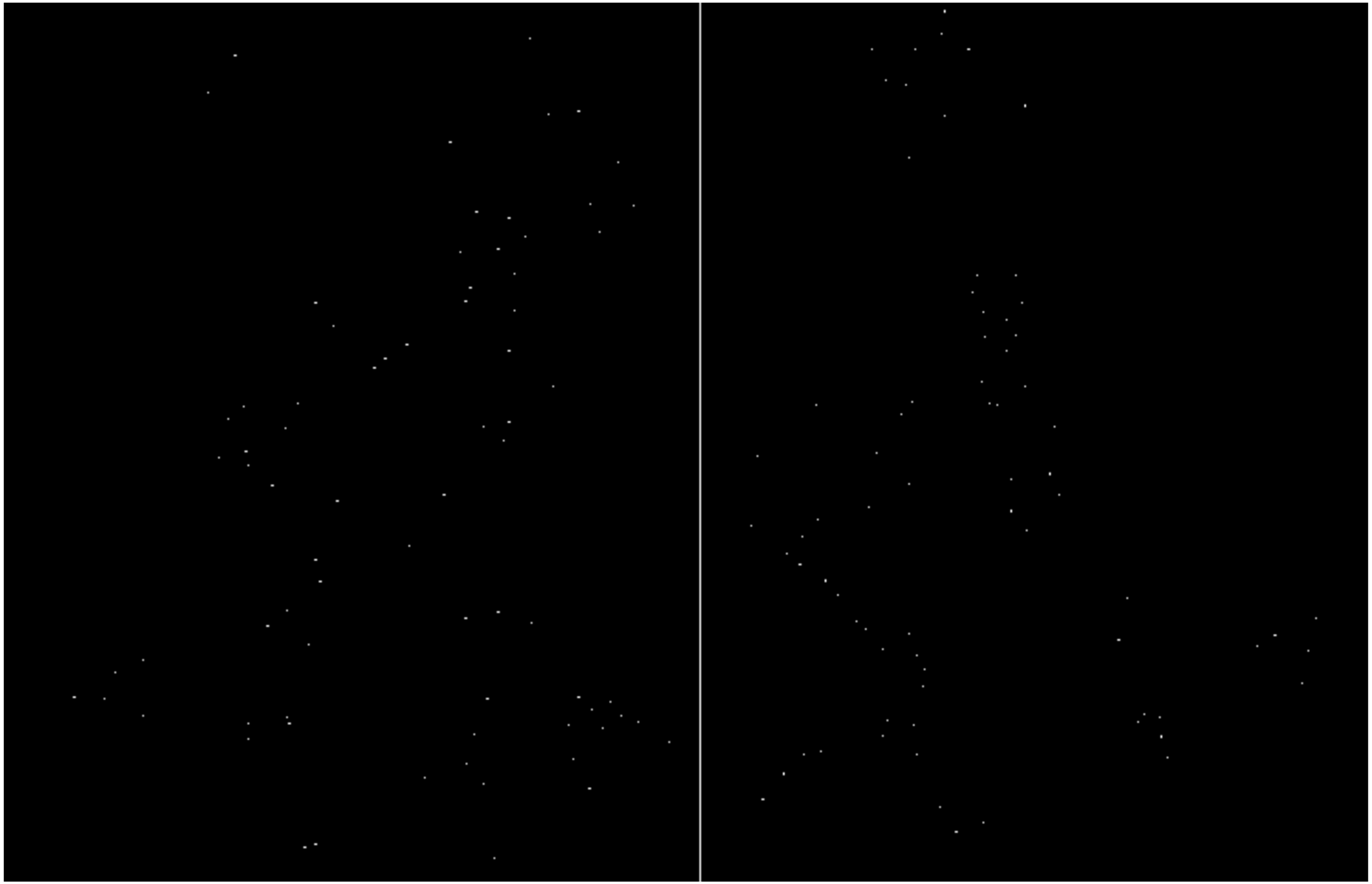
Harris Detector: Workflow

Find points with large corner response: R>threshold



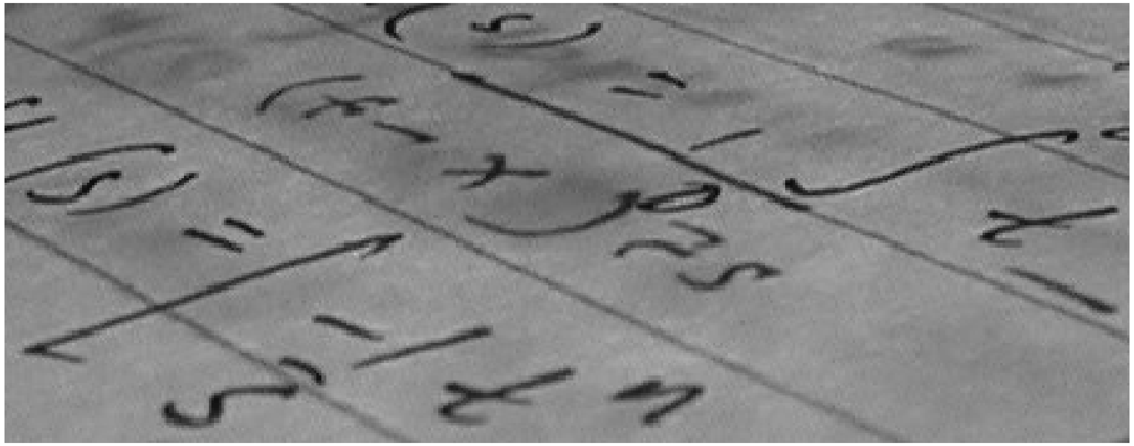
Harris Detector: Workflow

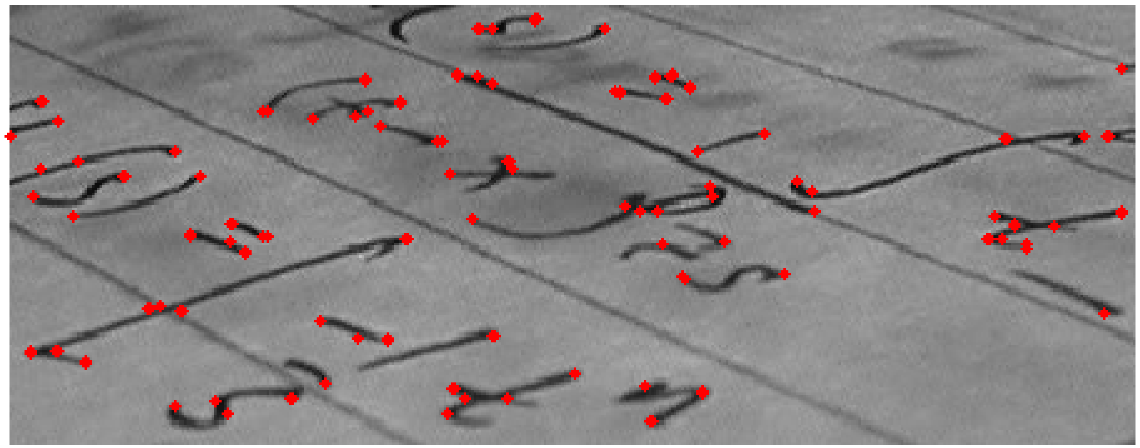
Take only the points of local maxima of R



Harris Detector: Workflow

Harris Detector: Example

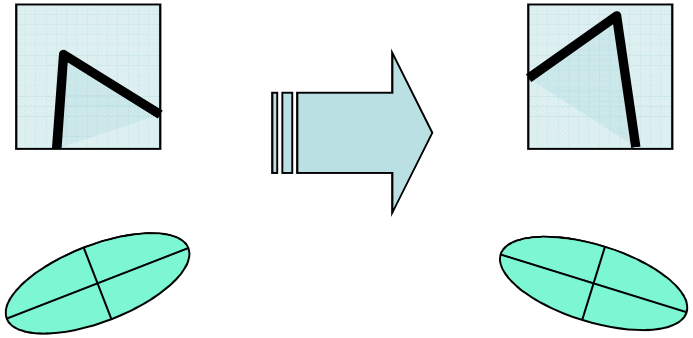




Harris Detector: Example

Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

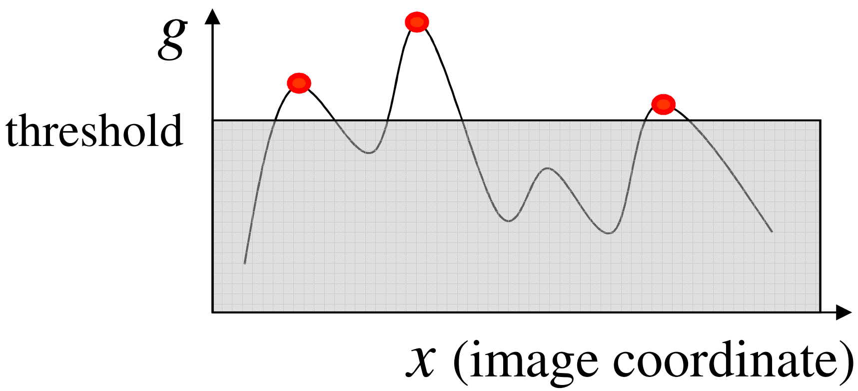
Corner response R is invariant to image rotation

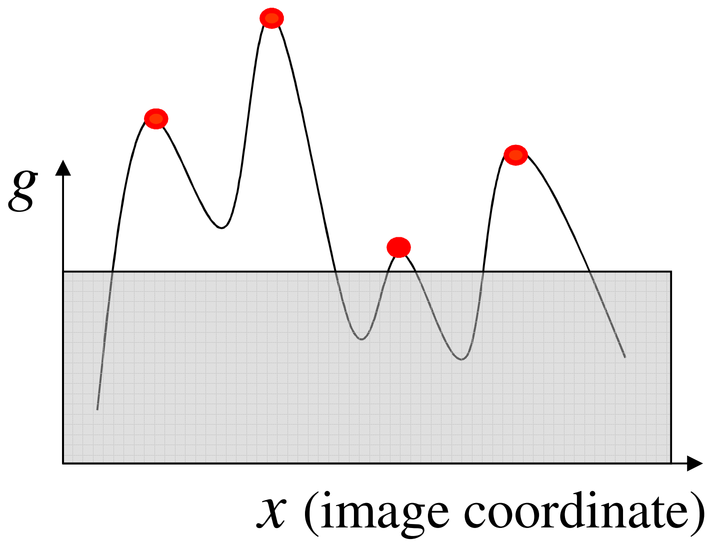
Harris Detector: Some Properties

Partial invariance to affine intensity change

✓Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

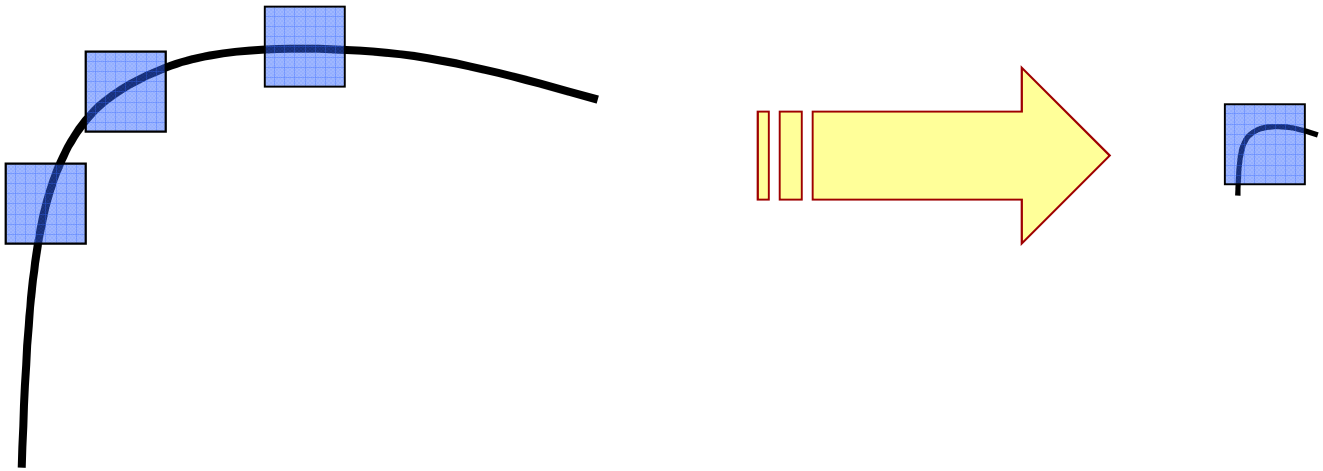
✓Intensity scale: $I \rightarrow a\, I$





Harris Detector: Some Properties

- But: non-invariant to spatial scale!

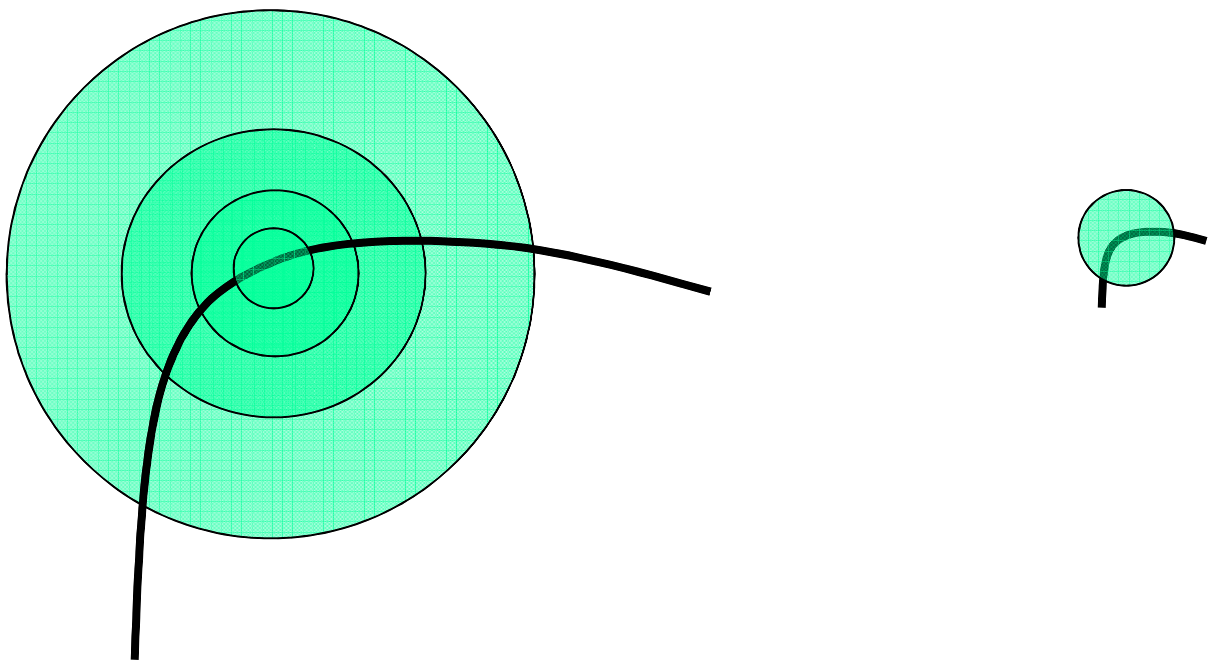


All points will be classified as edges

Corner !

Scale Invariant Detection

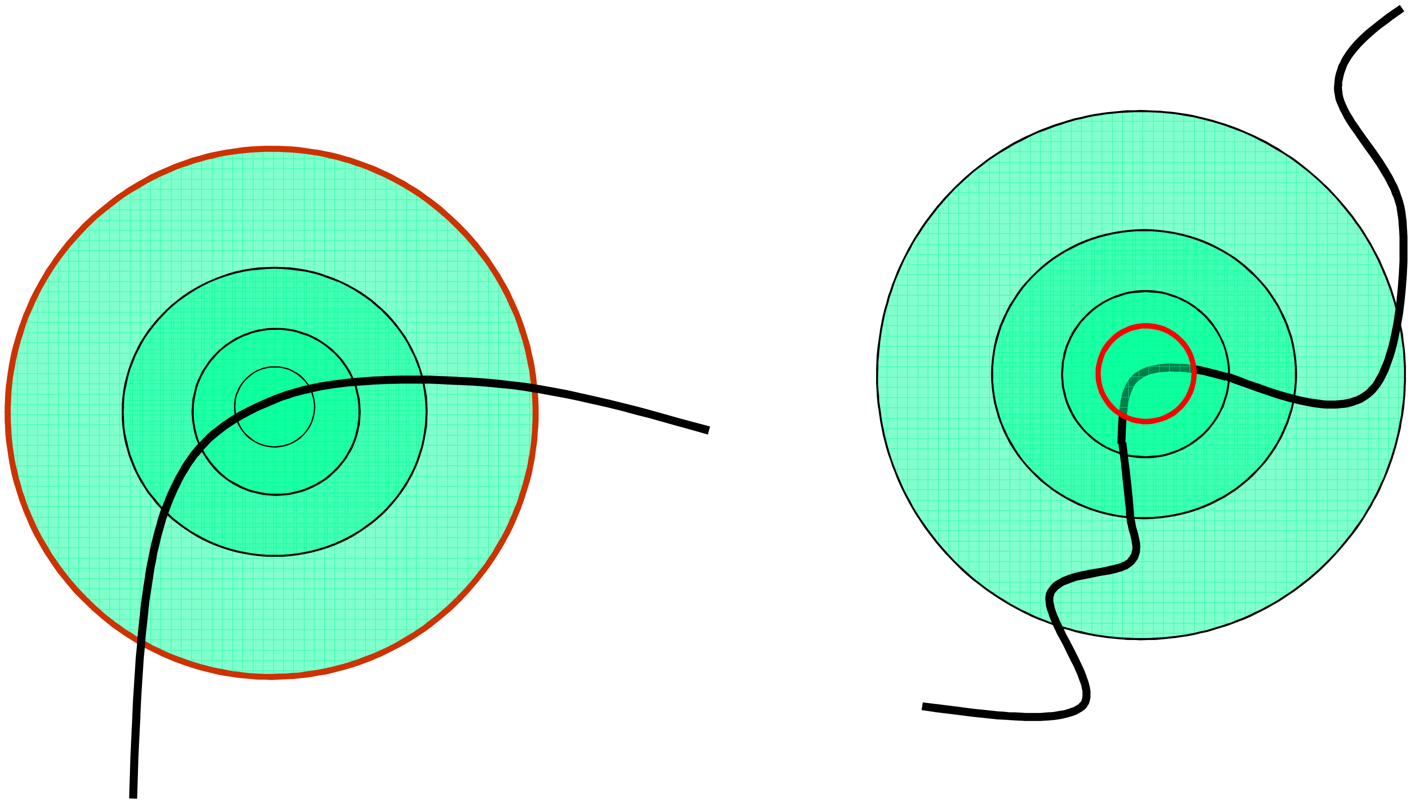
- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale Invariant Detection

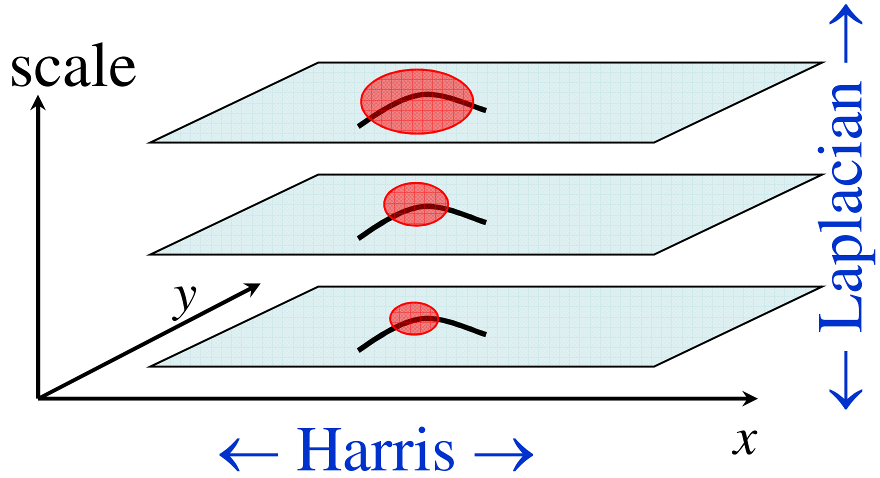
- **The problem**: how do we choose corresponding circles *independently* in each image?

- **Solution**: choose the scale of the "best" corner.



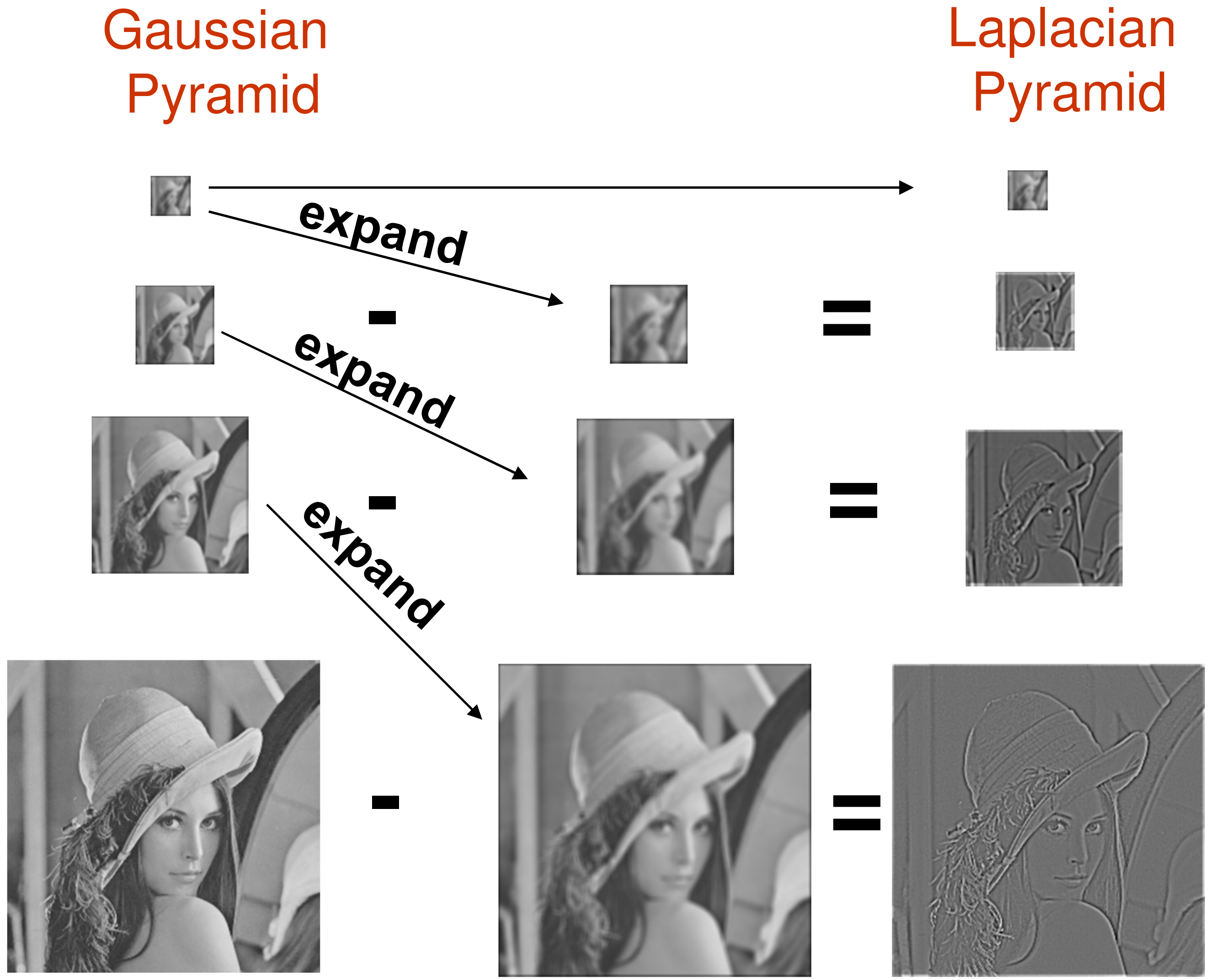
Harris-Laplacian Point Detector

- Harris-Laplacian
  Find local maximum of: Harris corner detector for a set of Laplacian images.



[1] K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001

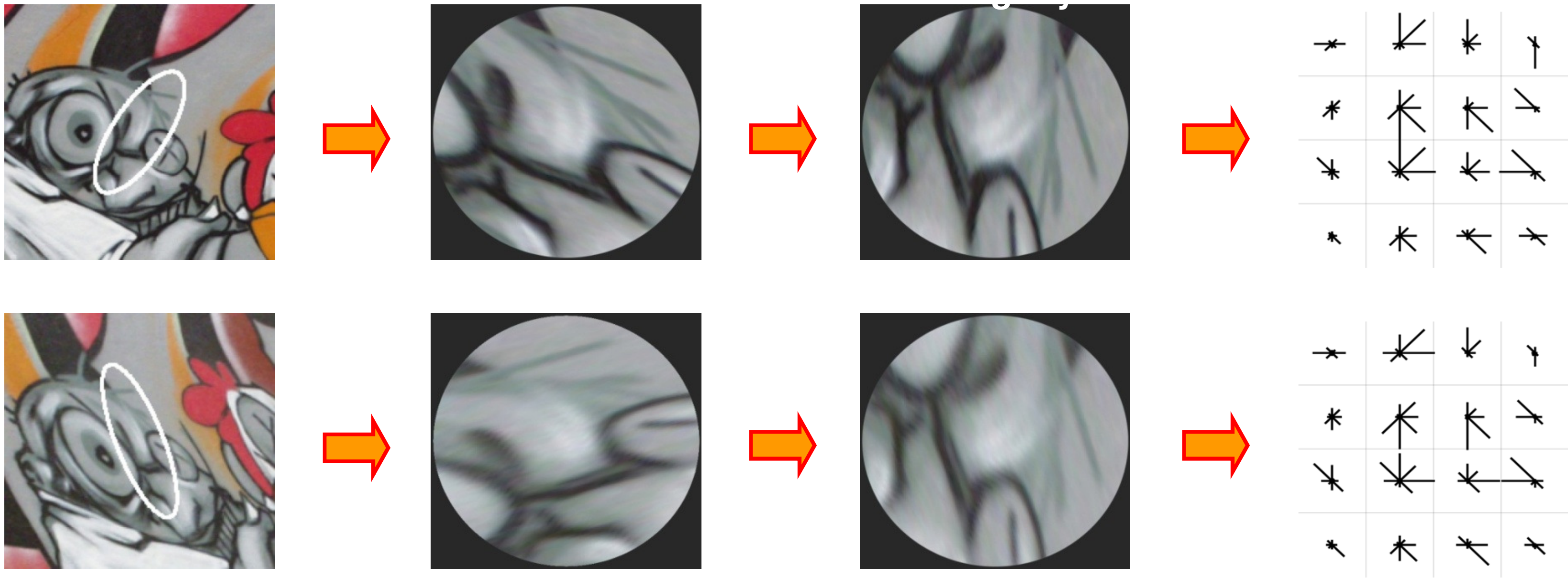
Memo: Gaussian / Laplacian Pyramids



Harris - Laplacian Detector

SIFT – Scale Invariant Feature Transform

David G. Lowe, "Distinctive image features from scale-invariant keypoints", International Journal of Computer Vision, 60, 2 (2004), pp. 91-110



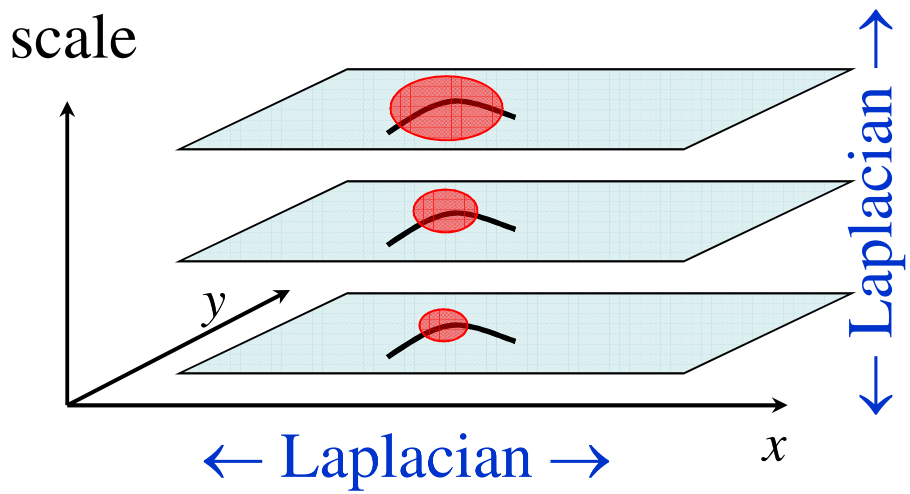
SIFT – Scale Invariant Feature Transform

David G. Lowe, "Distinctive image features from scale-invariant keypoints", International Journal of Computer Vision, 60, 2 (2004), pp. 91-110

- *Give about 2000 stable "keypoints" for a typical 500* x *500 image*
- *Each keypoint is described by a vector of*
  4 x 4 x 8 = 128 elements
  (over 4x4 array of 8-bin gradient histograms keypoint neighborhood)

SIFT – Scale Invariant Feature Transform

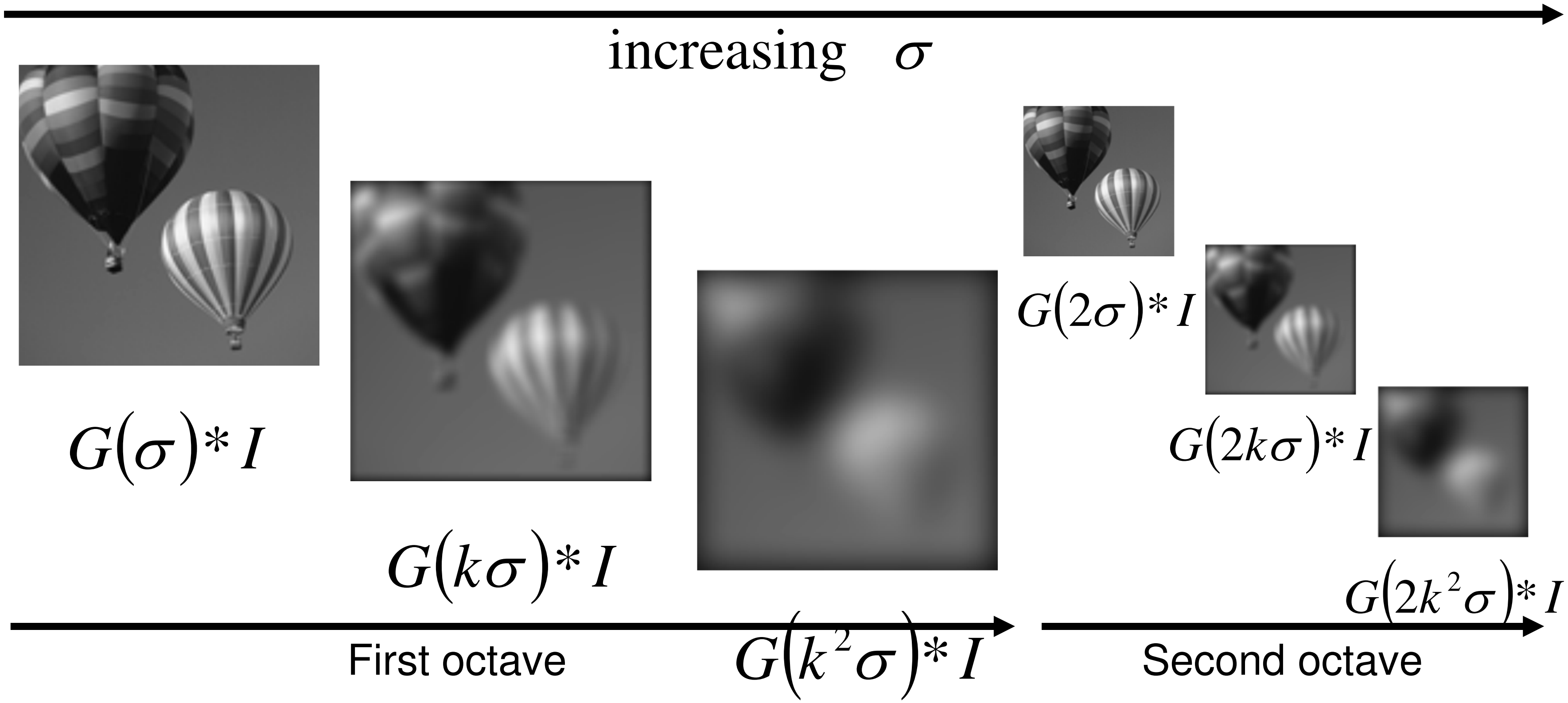
- *Find local maximum of* Laplacian in space and scale



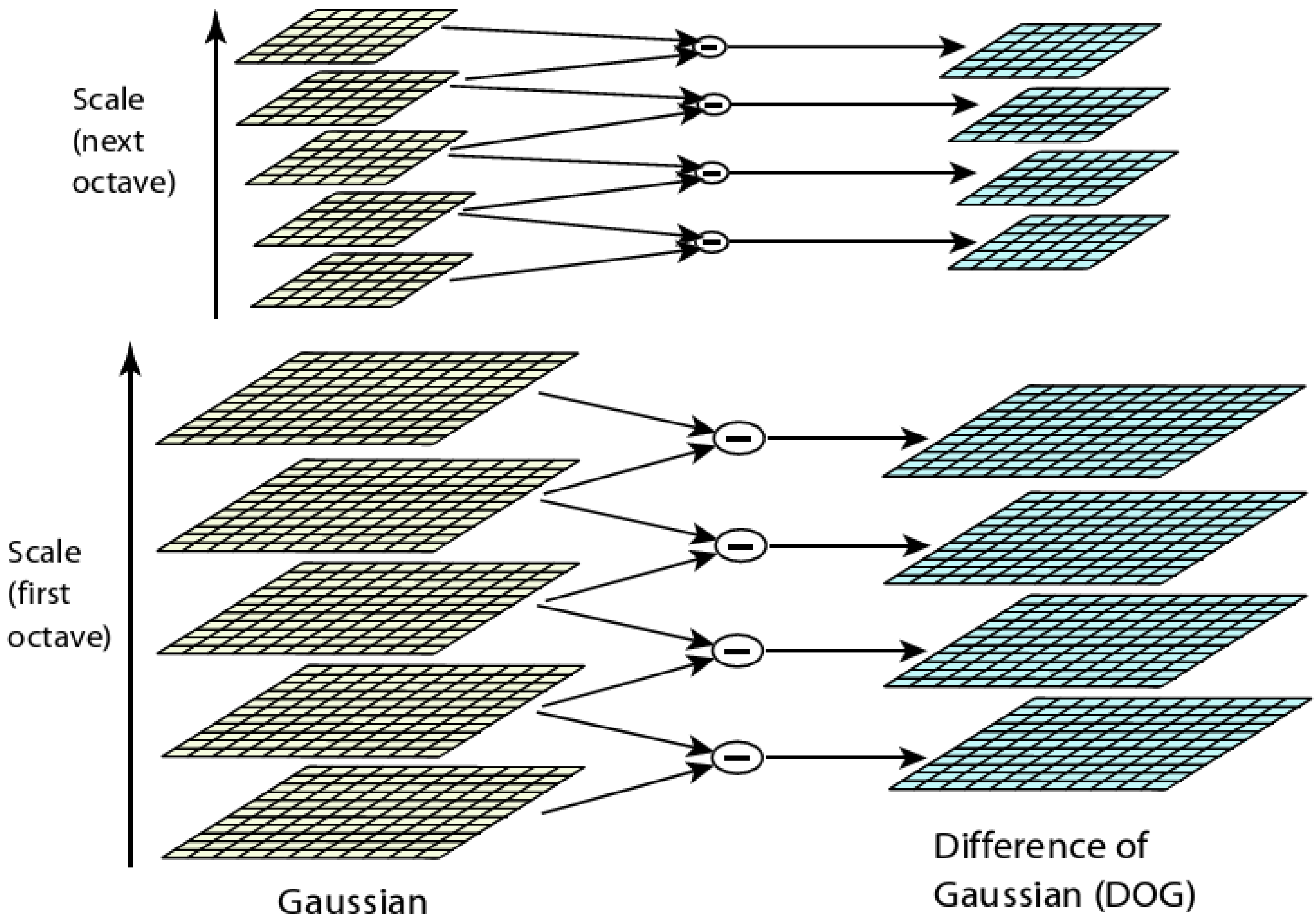
David G. Lowe, "Distinctive image features from scale-invariant keypoints", International Journal of Computer Vision, 60, 2 (2004), pp. 91-110

SIFT - Point Detection

- Construct scale-space:



SIFT – Scale Space

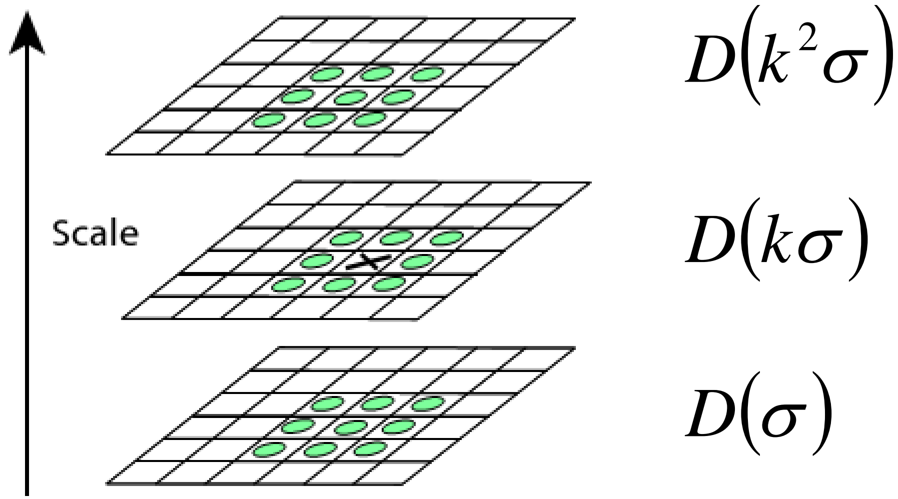


SIFT – point detection

STEP 1:

Determine local Maxima in DoG pyramid (Laplacian Pyramid).

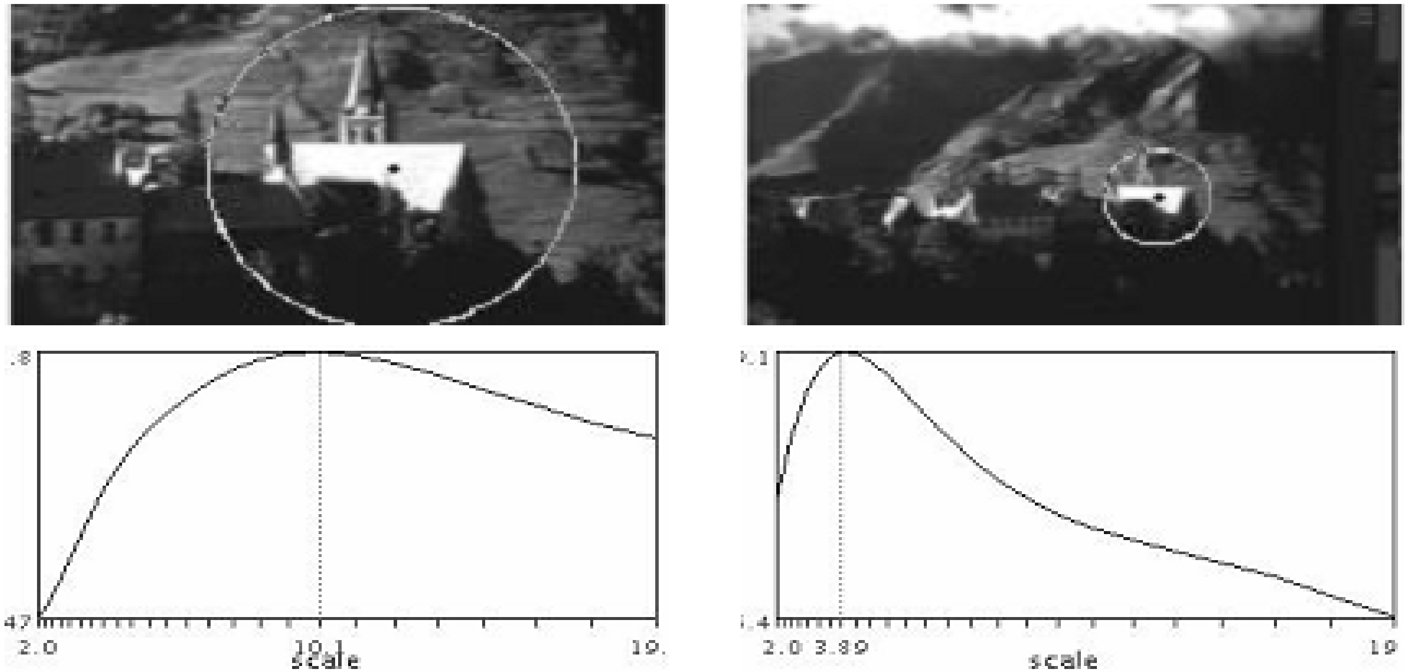
- Scale Space extrema detection.
- Choose all extrema within 3x3x3 neighborhood.



SIFT – point detection

STEP 1:

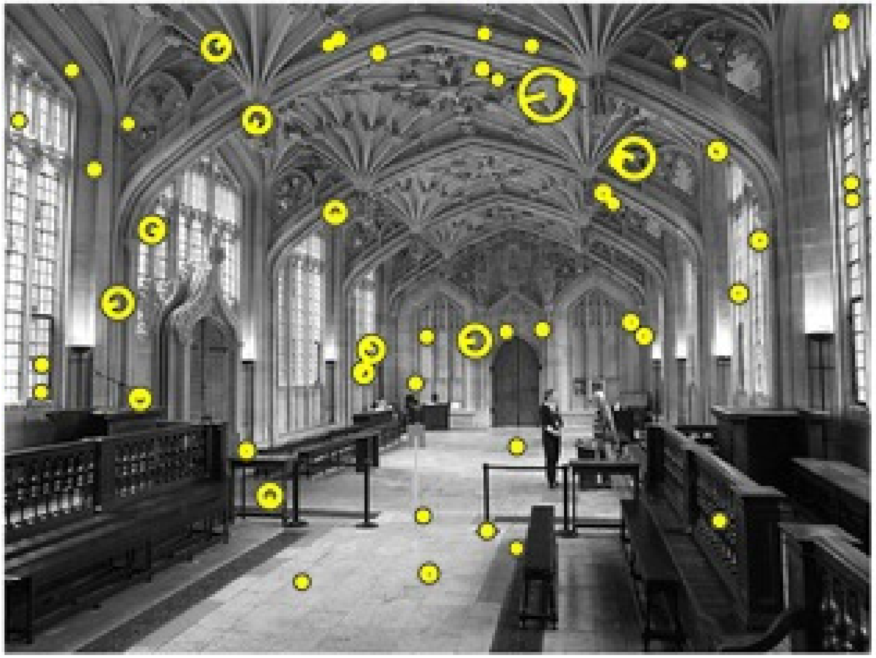
Determine local Maxima in DoG pyramid (Laplacian Pyramid).



Experimentally, Maximum of Laplacian gives best notion of sc

SIFT - Step 1: Interest Point Detection

Detections at multiple scales



Some of the detected SIFT frames.

http://www.vlfeat.org/overview/sift.html

SIFT – point detection

(a)

233x189 image

(b)

832 SIFT extrema

SIFT - Step 2: Interest Localization & Filtering

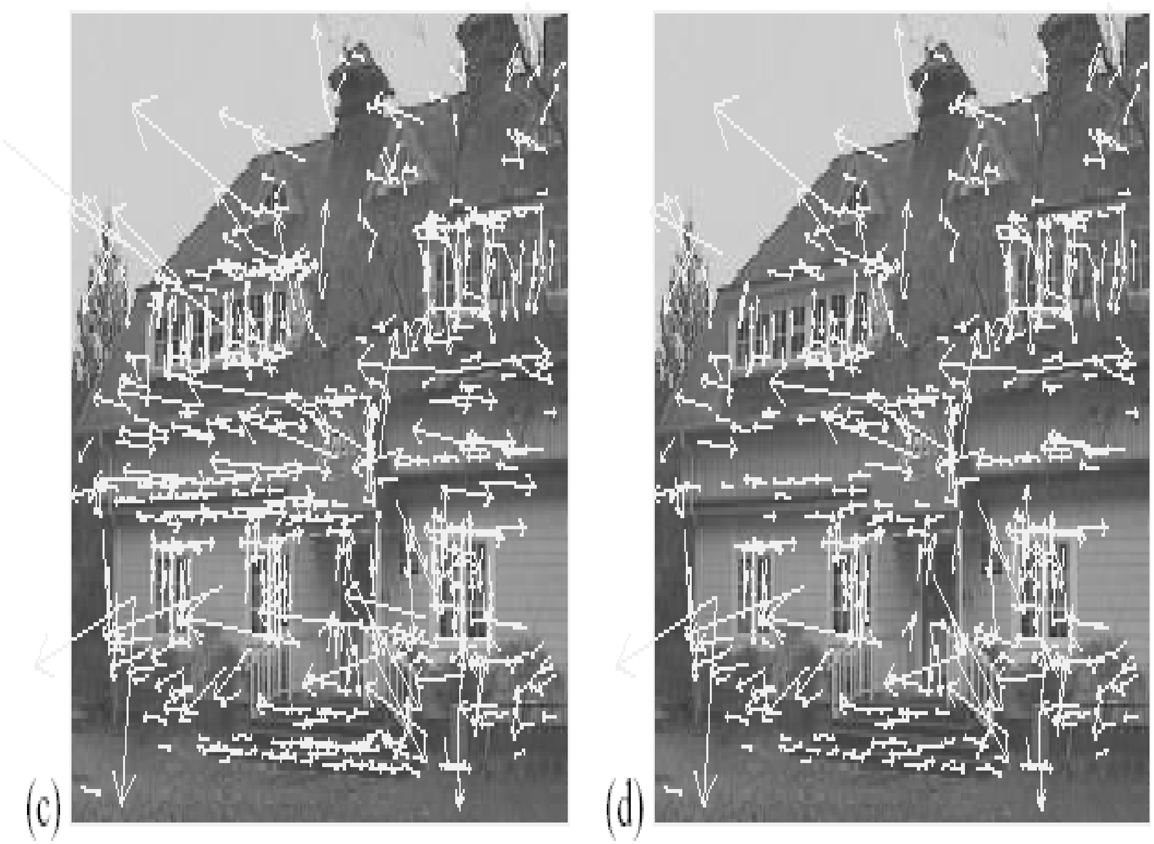
2) Remove bad Interest points:

a) Remove points with low contrast

b) Remove Edge points (Eigenvalues of Hessian Matrix must BOTH be large).

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

Interest Points

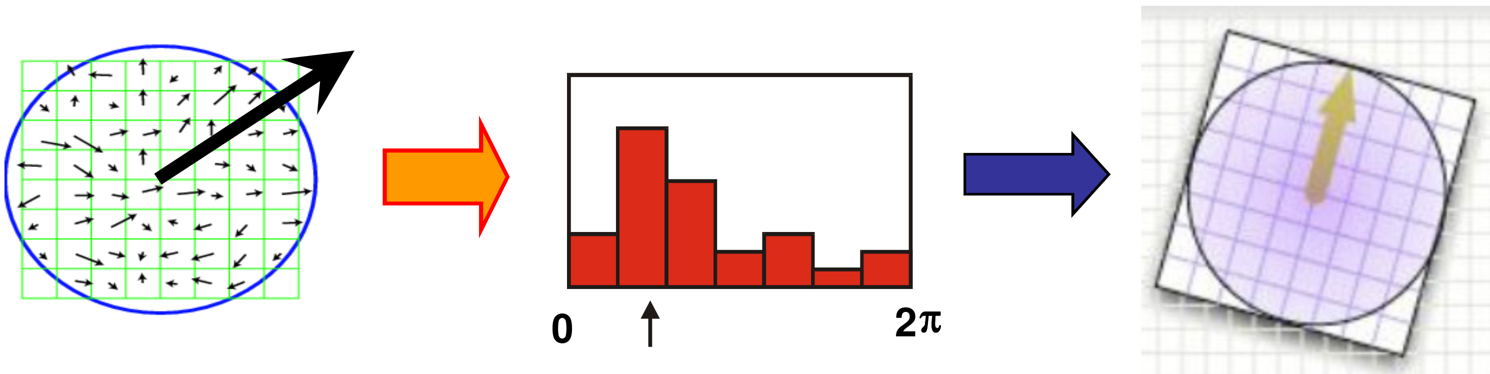


(c) 729 left after peak value threshold (from 832)
(d) 536 left after testing ratio of principle curvatures

SIFT – Descriptor Vector

STEP 3: Select canonical orientation

- Each SIFT interest point is associated with location (x,y) and scale (σ)
- Compute gradient magnitude and orientation for each SIFT point:

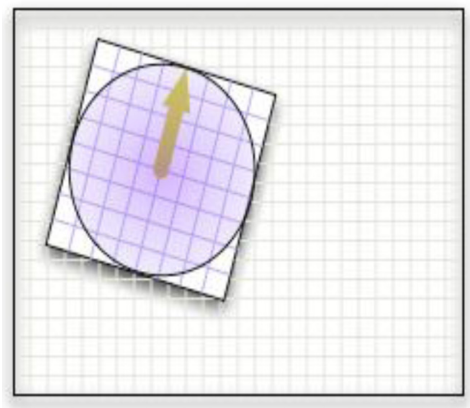


Assign canonical orientation at peak of smoothed histogram (fit parabola to better localize peak).

SIFT – Descriptor Vector

STEP 3: Select canonical orientation

- Each SIFT interest point is associated with location (x,y), scale (σ), gradient magnitude and orientation (m, θ).



- Compute SIFT feature - a vector of 128 entries.

SIFT – Descriptor Vector

STEP 4: Compute SIFT feature vector of 128 entries

- Gradients determined in 16x16 window at SIFT point in scale space.
- Histogram is computed for gradients of each 4x4 sub window in 8 **relative** directions.
- A 4x4x8 = 128 dimensional feature vector is produced.

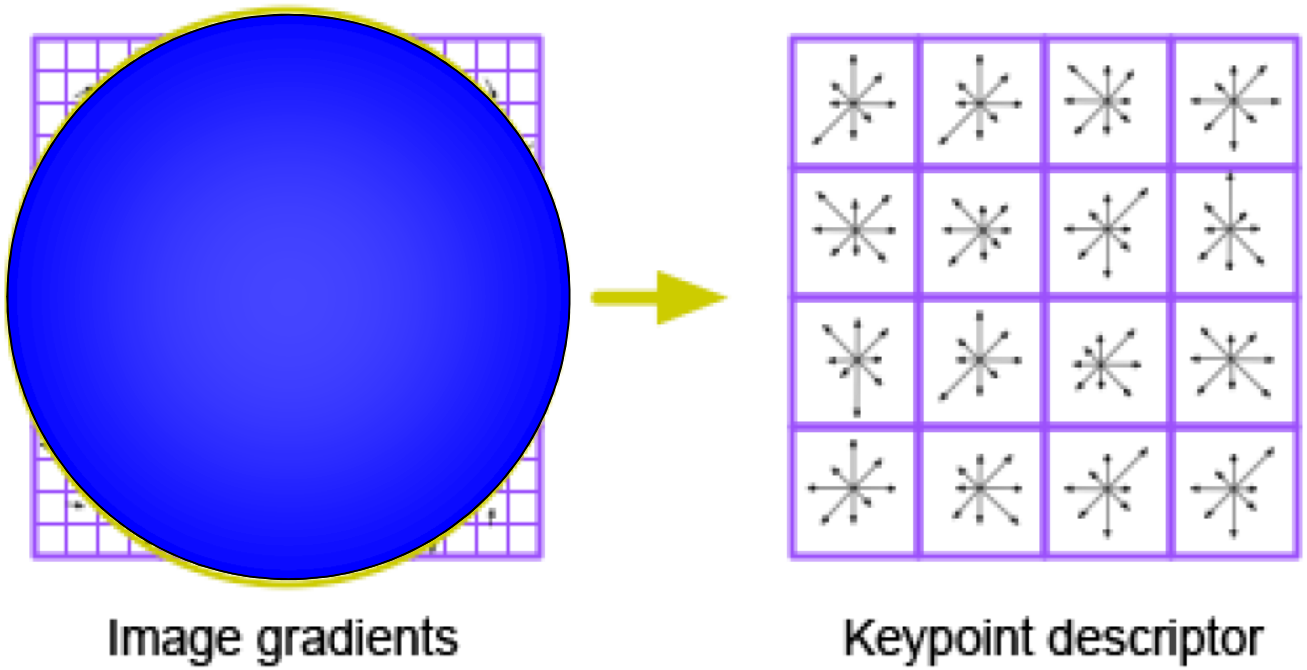
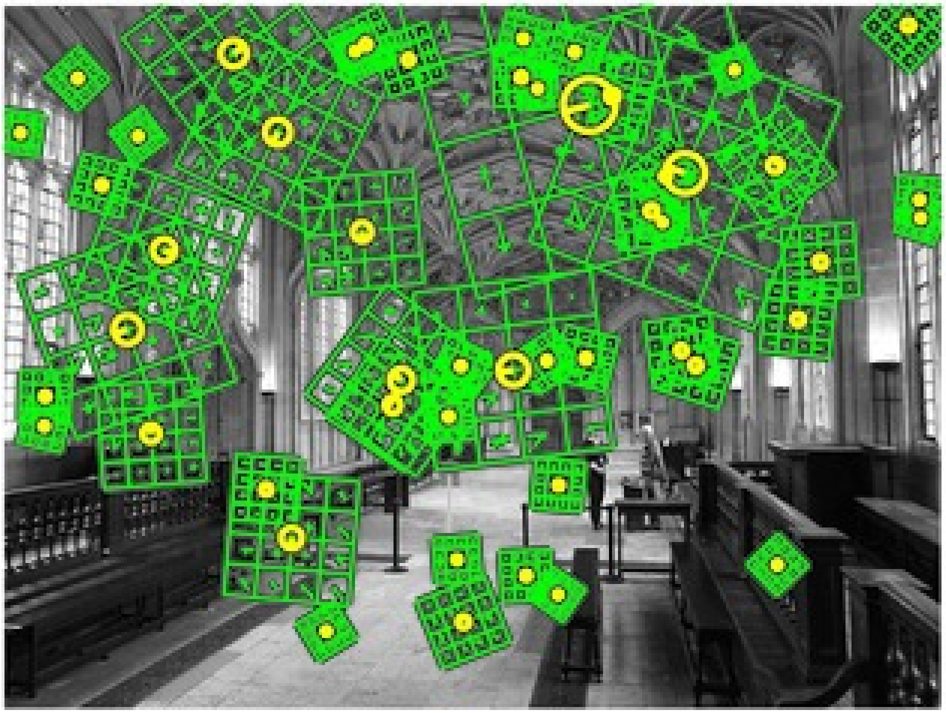


Image from: Jonas Hurrelmann

SIFT – Descriptor Vector

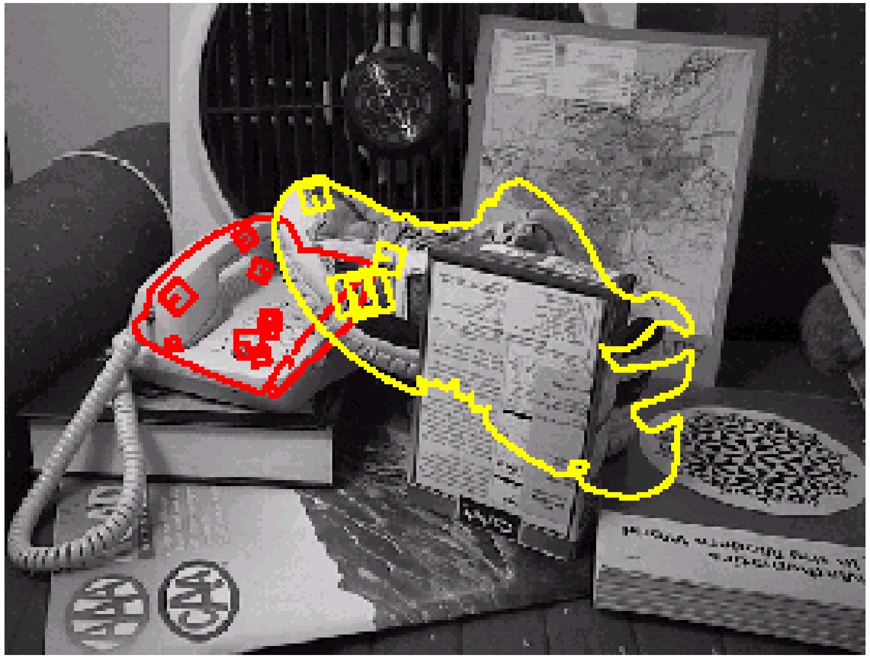
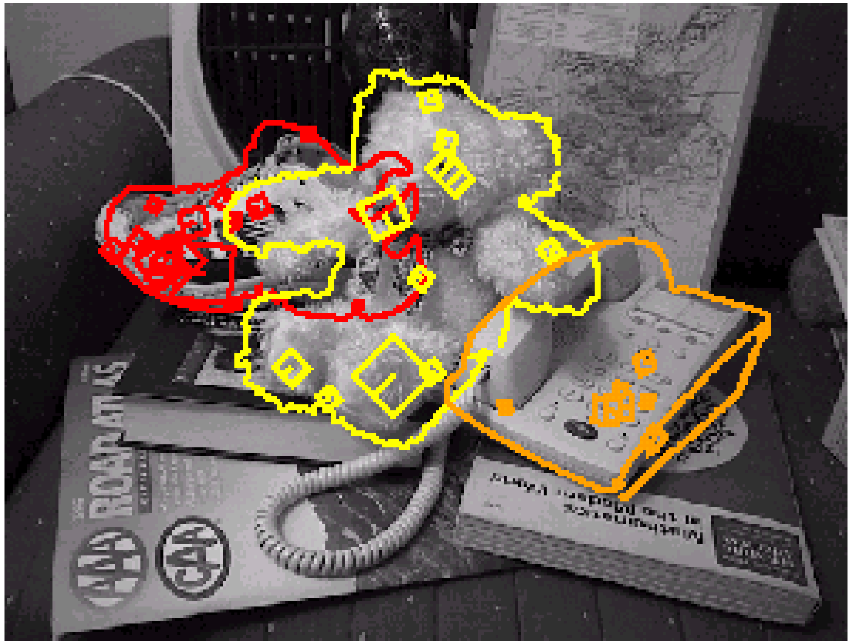
STEP 4: Compute feature vector



Object Recognition

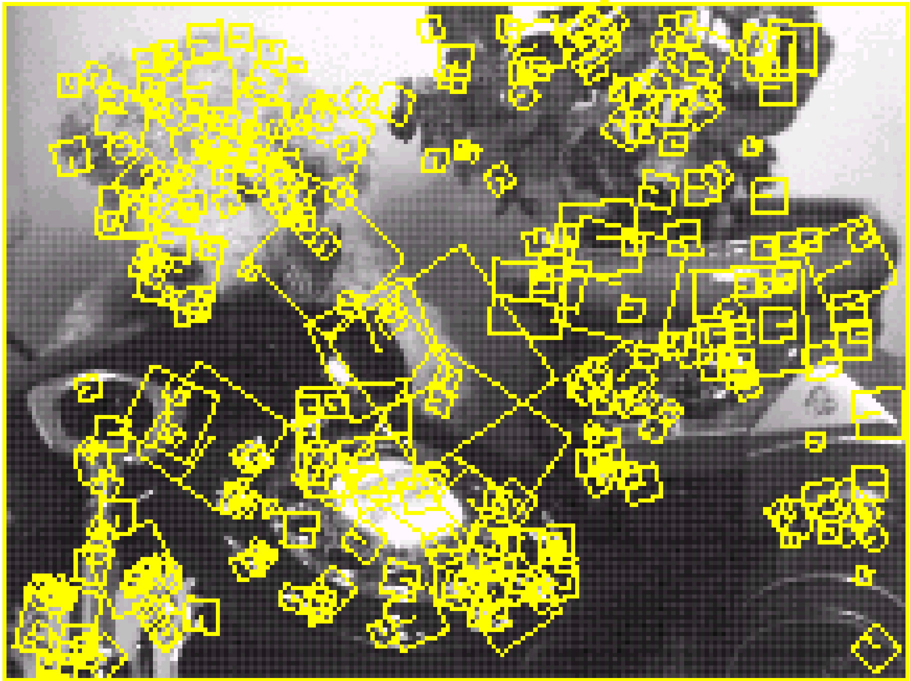
- Only 3 keys are needed for recognition, so extra keys provide robustness

Recognition under occlusion

Test of illumination Robustness

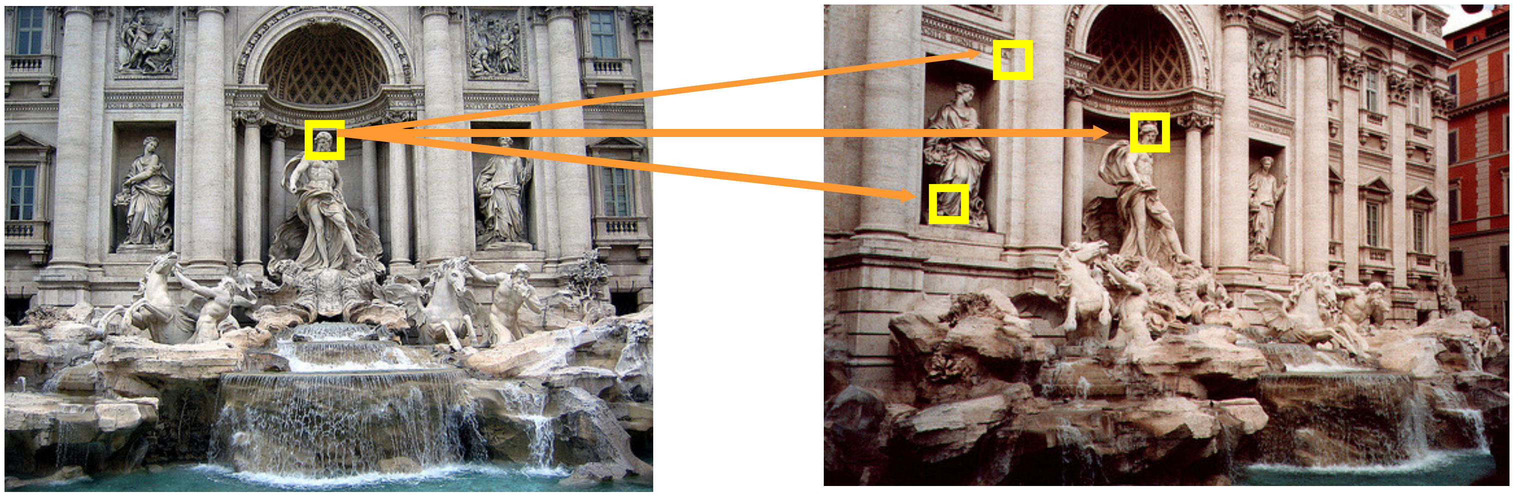
- Same **image** under differing illumination



273 keys verified in final match

Matching SIFT Features

- Given a feature in I_1, how to find the best match in I_2?
  1. Define distance function that compares two descriptors.
  2. Test all the features in I_2, find the one with min distance. Accept if below threshold.



I_1 I_2

Matching SIFT Features

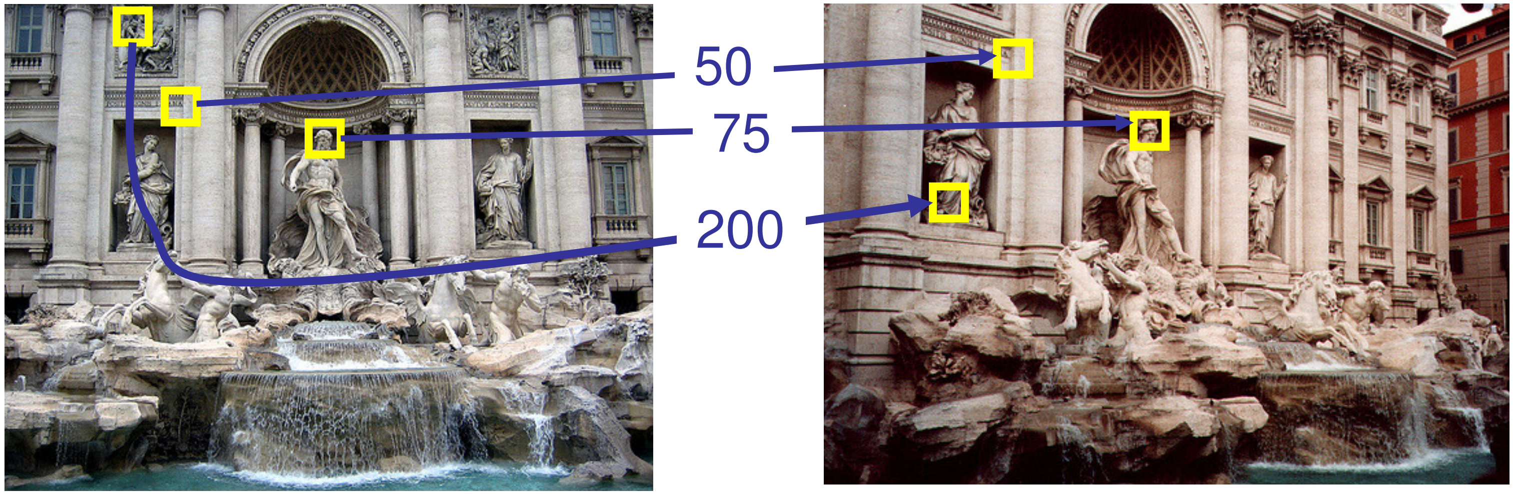
22 correct matches

Matching SIFT Features

33 correct matches

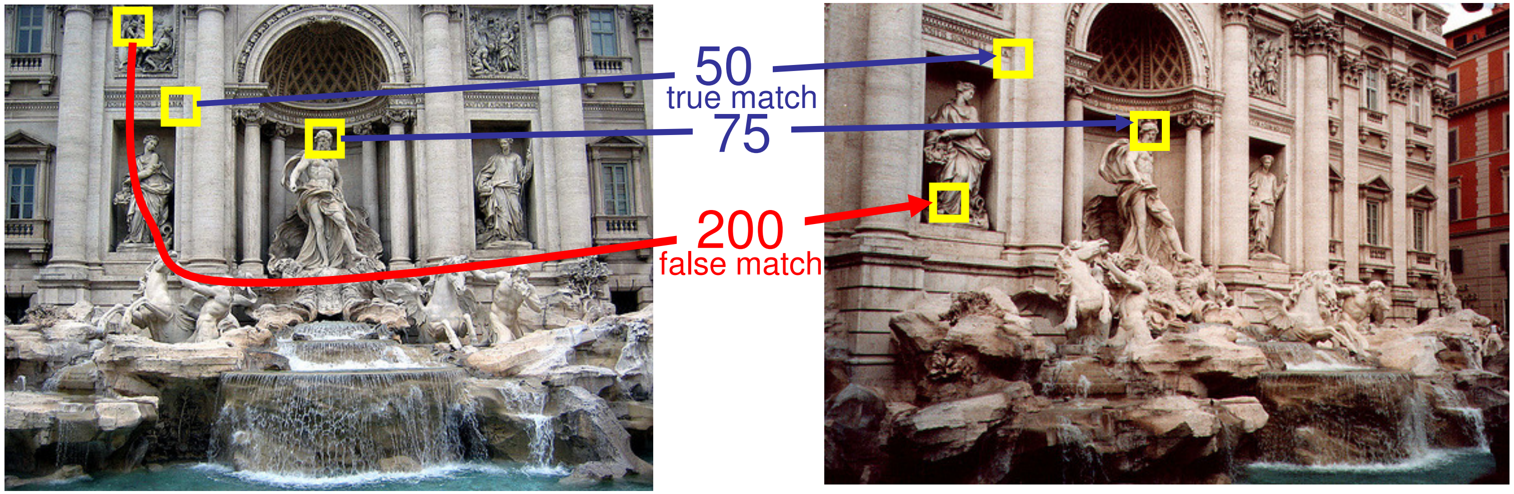
Matching SIFT Features

How to evaluate the performance of a feature matcher?



Matching SIFT Features

- Threshold t affects # of correct/false matches



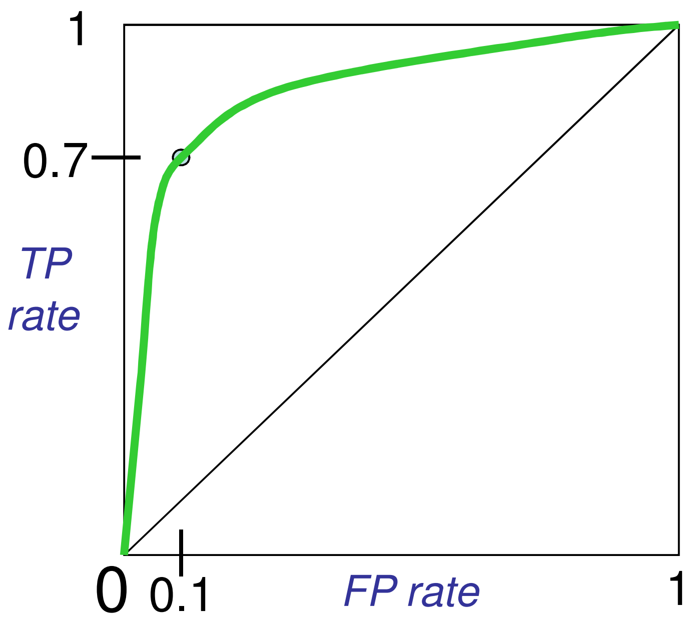
- True positives (TP) = # of detected matches that are correct
- False positives (FP) = # of detected matches that are incorrect

Matching SIFT Features

- ROC Curve

  - Generated by computing (FP, TP) for different thresholds.

  - Maximize area under the curve (AUC).

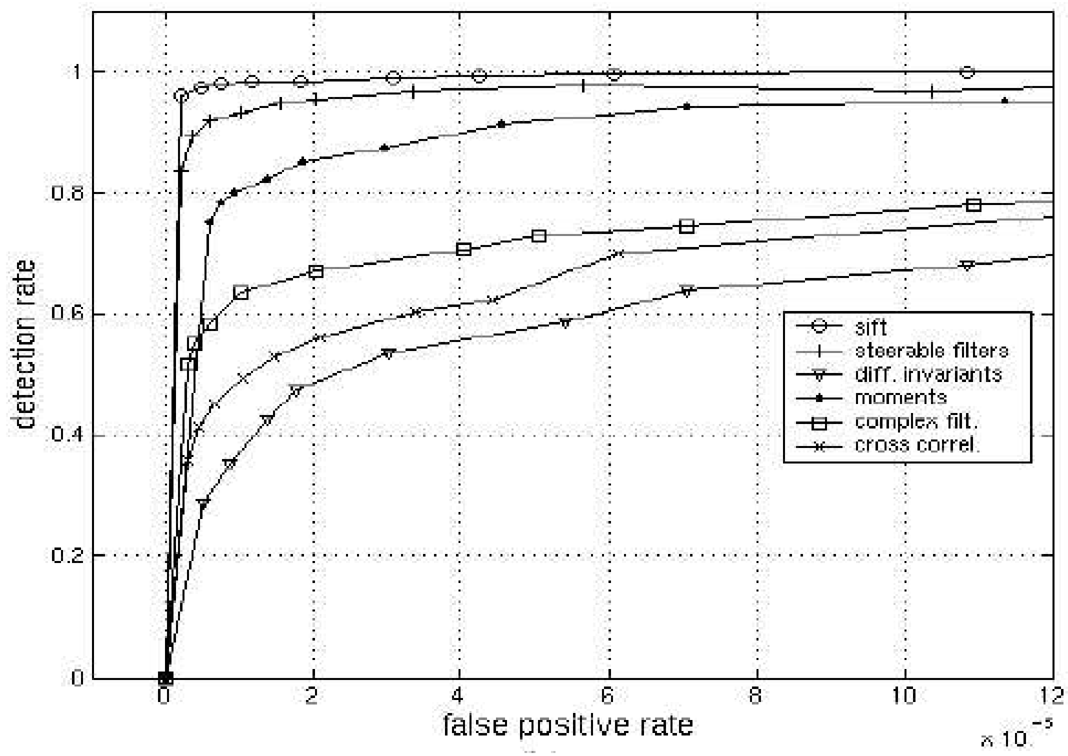


http://en.wikipedia.org/wiki/Receiver_operating_characteristic

Evaluating SIFT Features

- Empirically found[2] to show very good performance, invariant to *image rotation*, *scale*, *intensity change*, and to moderate *affine* transformations

Scale = 2.5
Rotation = 45^0



[1] D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". Accepted to IJCV 2004
[2] K.Mikolajczyk, C.Schmid. "A Performance Evaluation of Local Descriptors". CVPR 2003

Example - Mosaicing

Source: Alexei Efros

Example: Mosiacing (Panorama)

M. Brown and D. G. Lowe. Recognising Panoramas. ICCV 2003