

# Related-Key Rectangle Attack of the Full HAS-160 in Encryption Mode

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**Abstract.** In this paper we investigate the security of the compression function of HAS-160 in encryption mode. The structure of HAS-160 is similar to SHA-1 besides some modifications. This is the first cryptographic attack that breaks the encryption mode of the full 80-round HAS-160.

We apply a key recovery attack that requires  $2^{157}$  chosen plaintexts and  $2^{377.5}$  80-round HAS-160 encryptions. The attack does not aim for a collision, preimage or 2nd-preimage attack, but it shows that HAS-160's compression function used as a block cipher is not an ideal cipher.

**Keywords:** differential cryptanalysis, related-key rectangle attack, HAS-160.

## 1 Introduction

HAS-160 is a hash function that is widely used by the Korean industry, due to its standardization by the Korean government (TTAS.KO-12.0011/R1) [1]. Based on the MERKLE-DAMGÅRD structure [9, 17], it uses a compression function with input size of 512 bits and a chaining and output value of 160 bits. HAS-160 consists of a round function which is applied 80 times for each input message block. The overall design of the compression function is similar to the design of SHA-1 [18] and the MD family [19, 20], except a few modifications in the rotation constants and in the message expansion.

Up to now there are only a few cryptographic results on HAS-160. Yun et al. [24] found a collision on 45-round HAS-160 with complexity  $2^{12}$  by using the techniques introduced by Wang et al. [23]. Cho et al. [7] extended the previous result to break 53-round HAS-160 in time  $2^{55}$ . At ICISC 2007 Mendel and Rijmen [15] improved the attack complexity of the attack in [7] to  $2^{35}$  hash computations and they were able to present a colliding message pair for the 53-round version of HAS-160. They also showed how the attack can be extended to 59-round HAS-160 with a complexity of  $2^{55}$ .

HAS-160 in encryption mode is resistant to many attacks that can be applied to SHACAL-1, since its rotation constants are different and in each round its key schedule (which is equal to the message expansion) does not offer any

sliding properties. Nevertheless, it has a high degree of linearity which makes it vulnerable to related-key attacks.

In this paper we analyze the internal block cipher of HAS-160 and present the first cryptographic result on the full compression function of HAS-160. Using a related-key rectangle attack with four related keys we can break the full 80-rounds, i.e., recovering some key bits faster than exhaustive search. Our attack uses about  $2^{157}$  chosen plaintexts and runs in time of about  $2^{377.5}$  80-round HAS-160 encryptions. For comparison exhaustive key search would require about  $2^{512}$  80-round HAS-160 encryptions.

The paper is organized as follows: In Section 2 we give a brief description of the HAS-160 compression function in encryption mode. Section 3 discusses some crucial properties of HAS-160. In Section 4 we describe the related-key rectangle attack. Section 5 presents our related-key rectangle attack on the full HAS-160 encryption mode. Section 6 concludes the paper.

## 2 Description of the HAS-160 Encryption Mode

### 2.1 Notation

The following notations are used in this paper:

- $\oplus$  : bitwise XOR operation
- $\wedge$  : bitwise AND operation
- $\vee$  : bitwise OR operation
- $X \lll^k$  : bit-rotation of  $X$  by  $k$  positions to the left.
- $\boxplus$  : addition modulo  $2^{32}$  operation
- $\neg$  : bitwise complement operation
- $e_i$  : a 32-bit word with zeros in all positions except for bit  $i$ , ( $0 \leq i \leq 31$ )
- $e_{i_1, \dots, i_l} : e_{i_1} \oplus \dots \oplus e_{i_l}$

The bit positions follow the little endian convention, i.e., for a 32-bit word bit 31 is the most significant bit and bit 0 is the least significant bit.

### 2.2 HAS-160

Now we describe the structure of HAS-160 and how it can be used as a block cipher. The inner block cipher operates on a 160-bit message block and a 512-bit master key. A 160-bit plaintext  $P_0 = A_0 || B_0 || C_0 || D_0 || E_0$  is divided into five 32-bit words  $A_0, B_0, C_0, D_0, E_0$ . HAS-160 consists of 4 passes of 20 rounds each, i.e., the round function is applied 80 times in total. The corresponding ciphertext,  $P_{80}$ , is denoted by  $A_{80} || B_{80} || C_{80} || D_{80} || E_{80}$ . The round function at round  $i$  ( $i = 1, \dots, 80$ ) can be described as follows:

$$\begin{aligned}
A_i &\leftarrow A_{i-1} \lll^{s_{1,i}} \boxplus f_i(B_{i-1}, C_{i-1}, D_{i-1}) \boxplus E_{i-1} \boxplus k_i + c_i, \\
B_i &\leftarrow A_{i-1}, \\
C_i &\leftarrow B_{i-1} \lll^{s_{2,i}}, \\
D_i &\leftarrow C_{i-1}, \\
E_i &\leftarrow D_{i-1},
\end{aligned}$$

where  $c_i$  and  $k_i$  represents the  $i$ -th round constant and the  $i$ -th round key respectively, while  $f_i(\cdot)$  represents a boolean function. The function  $f_i(\cdot)$  and the constant  $c_i$  of round  $i$  can be found in Table 1.

**Table 1.** Boolean functions and constants

Pass	Round ( $i$ )	Boolean function ( $f_i$ )	Constant ( $c_i$ )
1	1 – 20	$(x \wedge y) \vee (\neg x \wedge z)$	0x00000000
2	21 – 40	$x \oplus y \oplus z$	0x5a827999
3	41 – 60	$(x \vee \neg z) \oplus y$	0x6ed9eba1
4	61 – 80	$x \oplus y \oplus z$	0x8f1bbcdc

The rotation constant  $s_{1,i}$  used in round  $i$  are given in Table 2.

**Table 2.** The bit rotation  $s_1$

Round $(i \bmod 20) + 1$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$s_{1,i}$	13	5	11	7	15	6	13	8	14	7	12	9	11	8	15	6	12	9	14	5

The rotation constant  $s_{2,i}$  depends on the pass, i.e., it changes the value if the pass is changed but it is constant in each pass. A pass contains 20 rounds of the encryption. The pass-dependent values of  $s_{2,i}$  are:

- Pass 1:  $s_{2,i} = 10$
- Pass 2:  $s_{2,i} = 17$
- Pass 3:  $s_{2,i} = 25$
- Pass 4:  $s_{2,i} = 30$

The 80 round keys  $k_i$ ,  $i \in \{1, 2, \dots, 80\}$  are derived from the master key  $K$ , which consists of sixteen 32-bit words  $K = x_0, x_1, \dots, x_{15}$ . The round keys  $k_i$  are obtained from the key schedule described in Table 3.

Figure 1 shows the round function of HAS-160.

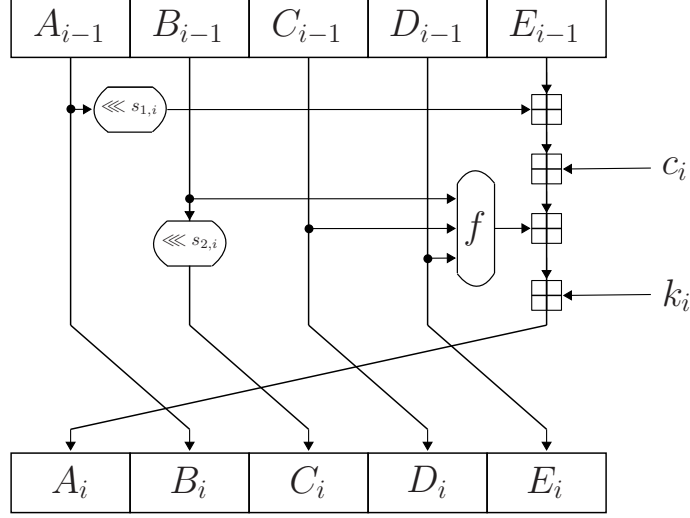
### 3 Properties of HAS-160

*Property 1.* (from [10]) Let  $Z = X \boxplus Y$  and  $Z^* = X^* \boxplus Y^*$  with  $X, Y, X^*, Y^*$  being 32-bit words. Then, the following properties hold:

1. If  $X \oplus X^* = e_j$  and  $Y = Y^*$ , then  $Z \oplus Z^* = e_{j,j+1,\dots,j+k-1}$  holds with probability  $2^{-k}$  for  $(j < 31, k \geq 1 \text{ and } j+k-1 \leq 30)$ . In addition, if  $j = 31$ ,  $Z \oplus Z^* = e_{31}$  holds with probability 1.
2. If  $X \oplus X^* = e_j$  and  $Y \oplus Y^* = e_j$ , then  $Z \oplus Z^* = e_{j+1,\dots,j+k-1}$  holds with probability  $2^{-k}$  for  $(j < 31, k \geq 1 \text{ and } j+k-1 \leq 30)$ . In addition, in case  $j = 31$ ,  $Z = Z^*$  holds with probability 1.

**Table 3.** The key schedule

Round $(i \bmod 20) + 1$	Pass 1	Pass 2	Pass 3	Pass 4
1	$x_8 \oplus x_9$ $\oplus x_{10} \oplus x_{11}$	$x_{11} \oplus x_{14}$ $\oplus x_1 \oplus x_4$	$x_4 \oplus x_{13}$ $\oplus x_6 \oplus x_{15}$	$x_{15} \oplus x_{10}$ $\oplus x_5 \oplus x_0$
2	$x_0$	$x_3$	$x_{12}$	$x_4$
3	$x_1$	$x_6$	$x_5$	$x_2$
4	$x_2$	$x_9$	$x_{14}$	$x_{13}$
5	$x_3$	$x_{12}$	$x_7$	$x_8$
6	$x_{12} \oplus x_{13}$ $\oplus x_{14} \oplus x_{15}$	$x_7 \oplus x_{10}$ $\oplus x_{13} \oplus x_0$	$x_8 \oplus x_1$ $\oplus x_{10} \oplus x_3$	$x_{11} \oplus x_6$ $\oplus x_1 \oplus x_{12}$
7	$x_4$	$x_{15}$	$x_0$	$x_3$
8	$x_5$	$x_2$	$x_9$	$x_{14}$
9	$x_6$	$x_5$	$x_2$	$x_9$
10	$x_7$	$x_8$	$x_{11}$	$x_4$
11	$x_0 \oplus x_1$ $\oplus x_2 \oplus x_3$	$x_3 \oplus x_6$ $\oplus x_9 \oplus x_{12}$	$x_{12} \oplus x_5$ $\oplus x_{14} \oplus x_7$	$x_7 \oplus x_2$ $\oplus x_{13} \oplus x_8$
12	$x_8$	$x_{11}$	$x_4$	$x_{15}$
13	$x_9$	$x_{14}$	$x_{13}$	$x_{10}$
14	$x_{10}$	$x_{14}$	$x_6$	$x_5$
15	$x_{11}$	$x_4$	$x_{15}$	$x_0$
16	$x_4 \oplus x_5$ $\oplus x_6 \oplus x_7$	$x_{15} \oplus x_2$ $\oplus x_5 \oplus x_8$	$x_0 \oplus x_9$ $\oplus x_2 \oplus x_{11}$	$x_3 \oplus x_{14}$ $\oplus x_9 \oplus x_4$
17	$x_{12}$	$x_7$	$x_8$	$x_{11}$
18	$x_{13}$	$x_{10}$	$x_1$	$x_6$
19	$x_{14}$	$x_{13}$	$x_{10}$	$x_1$
20	$x_{15}$	$x_0$	$x_3$	$x_{12}$



**Fig. 1.** The round function of HAS-160

A more general description of these properties can be derived from the following theorem.

**Theorem 1.** (from [14]) Given three 32-bit XOR differences  $\Delta X, \Delta Y$  and  $\Delta Z$ . If the probability  $\Pr[(\Delta X, \Delta Y) \stackrel{\boxplus}{\rightarrow} \Delta Z] > 0$ , then

$$\Pr[(\Delta X, \Delta Y) \stackrel{\boxplus}{\rightarrow} \Delta Z] = 2^{-k},$$

where the integer  $k$  is given by  $k = \#\{i | 0 \leq i \leq 30, \text{ not } ((\Delta X)_i = (\Delta Y)_i = (\Delta Z)_i)\}$ .

*Property 2.* Consider the difference  $\Delta P_i = (\Delta A_i, \Delta B_i, \Delta C_i, \Delta D_i, \Delta E_i)$  of a message pair in round  $i$ . Then we know some 32-bit differences in round  $i + 1$ ,  $i + 2$ ,  $i + 3$  and  $i + 4$ . The known word differences are as follows:

$$\begin{aligned} (\Delta B_{i+1}, \Delta C_{i+1}, \Delta D_{i+1}, \Delta E_{i+1}) &= (\Delta A_i, \Delta B_i \lll s_{2,i+1}, \Delta C_i, \Delta D_i), \\ (\Delta C_{i+2}, \Delta D_{i+2}, \Delta E_{i+2}) &= (\Delta A_i \lll s_{2,i+2}, \Delta B_i \lll s_{2,i+1}, \Delta C_i), \\ (\Delta D_{i+3}, \Delta E_{i+3}) &= (\Delta A_i \lll s_{2,i+2}, \Delta B_i \lll s_{2,i+1}), \\ (\Delta E_{i+4}) &= (\Delta A_i \lll s_{2,i+2}) \end{aligned}$$

## 4 The Related-Key Rectangle Attack

The boomerang attack [21] is an extension to differential cryptanalysis [5] using adaptive chosen plaintexts and ciphertexts to attack block ciphers. The amplified boomerang attack [12] transforms the ordinary boomerang attack into a chosen

plaintext attack. A more detailed analysis along some improvements resulted in the rectangle attack [3]. The related-key rectangle attack is published in [4, 11, 13]. It is an adaptation of the rectangle attack into the related-key framework [2]. The attack can be described as follows.

A block cipher  $E : \{0, 1\}^n \times \{0, 1\}^k \rightarrow \{0, 1\}^n$  with  $E_K(\cdot) := E(K, \cdot)$  is treated as a cascade of two sub-ciphers  $E_{K^i}(P) = E1_{K^i}(E0_{K^i}(P))$ , where  $P$  is a plaintext encrypted under the key  $K^i$ . It is assumed that there exists a related-key differential  $\alpha \rightarrow \beta$  which holds with probability  $p$  for  $E0$ , i.e.,  $\Pr[E0_{K^a}(P^a) \oplus E0_{K^b}(P^b) = \beta | P^a \oplus P^b = \alpha] = p$ , where  $K^a$  and  $K^b = K^a \oplus \Delta K^*$  are two related keys and  $\Delta K^*$  is a known key difference (the same holds for  $\Pr[E0_{K^c}(P^c) \oplus E0_{K^d}(P^d) = \beta | P^c \oplus P^d = \alpha] = p$ , where  $K^c$  and  $K^d = K^c \oplus \Delta K^*$  are two related keys). Let  $X^i = E0_{K^i}(P^i)$ ,  $i \in \{a, b, c, d\}$  be an intermediate encryption value. We assume that there is a related-key differential  $\gamma \rightarrow \delta$  which holds with probability  $q$  for  $E1$ , i.e.,  $\Pr[E1_{K^a}(X^a) \oplus E1_{K^c}(X^c) = \delta | X^a \oplus X^c = \gamma] = q$ , where the keys  $K^a$  and  $K^c$  are related as  $K^a \oplus K^c = \Delta K'$  and  $\Delta K'$  is a known key difference (the same holds for  $\Pr[E1_{K^b}(X^b) \oplus E1_{K^d}(X^d) = \delta | X^b \oplus X^d = \gamma] = q$  where the keys  $K^b$  and  $K^d$  are related as  $K^b \oplus K^d = \Delta K'$ ). In our attack we use four different keys but one can also apply the attack with more or less keys.

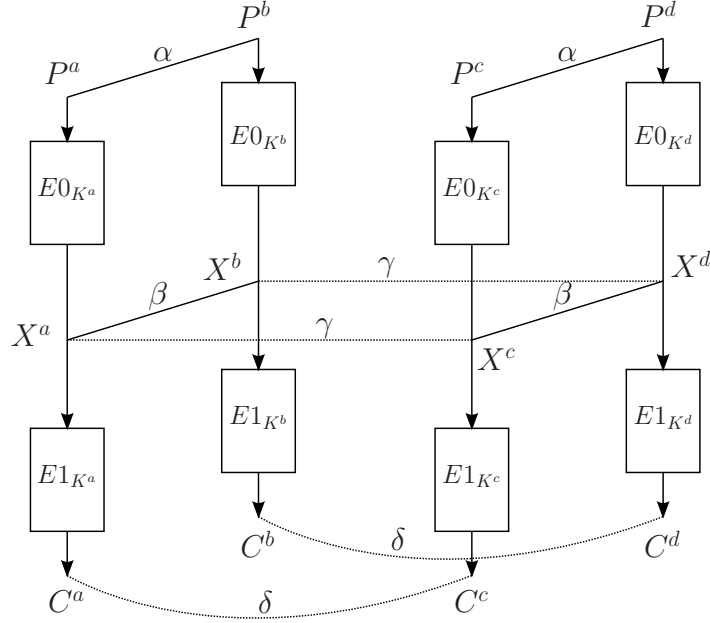
We discuss plaintext quartets  $(P^a, P^b, P^c, P^d)$ , for which  $P^a \oplus P^b = \alpha = P^c \oplus P^d$ , where  $P^i$  is encrypted under the key  $K^i$ ,  $i \in \{a, b, c, d\}$ . Out of  $N$  pairs of plaintexts with the related-key difference  $\alpha$ , about  $N \cdot p$  pairs have an output difference  $\beta$  after  $E0$ . These pairs can be combined into  $(N \cdot p)^2$  quartets, such that each quartet satisfies  $E0_{K^a}(P^a) \oplus E0_{K^b}(P^b) = \beta$  and  $E0_{K^c}(P^c) \oplus E0_{K^d}(P^d) = \beta$ . We assume that the intermediate values after  $E0$  distribute uniformly over all possible values. Thus,  $E0_{K^a}(P^a) \oplus E0_{K^c}(P^c) = \gamma$  holds with probability  $2^{-n}$ . If this occurs,  $E0_{K^b}(P^b) \oplus E0_{K^d}(P^d) = \gamma$  holds as well, since the following condition holds:

$$\begin{aligned} (E0_{K^a}(P^a) \oplus E0_{K^b}(P^b)) \oplus (E0_{K^c}(P^c) \oplus E0_{K^d}(P^d)) \\ \oplus (E0_{K^a}(P^a) \oplus E0_{K^c}(P^c)) = \\ (X^a \oplus X^b) \oplus (X^c \oplus X^d) \oplus (X^a \oplus X^c) = \\ \beta \oplus \beta \oplus \gamma = \gamma \end{aligned}$$

The expected number of quartets satisfying both  $E1_{K^a}(X^a) \oplus E1_{K^c}(X^c) = \delta$  and  $E1_{K^b}(X^b) \oplus E1_{K^d}(X^d) = \delta$  is

$$\sum_{\beta, \gamma} (N \cdot p)^2 \cdot 2^{-n} \cdot q^2 = N^2 \cdot 2^{-n} \cdot (p \cdot q)^2.$$

For a random cipher, the expected number of correct quartets is about  $N^2 \cdot 2^{-2n}$ . Therefore, if  $p \cdot q > 2^{-n/2}$  and  $N$  is sufficiently large, the related-key rectangle distinguisher can distinguish between  $E$  and a random cipher. Figure 2 visualizes the structure of the related-key rectangle distinguisher.



**Fig. 2.** The related-key rectangle distinguisher

## 5 Related-Key Rectangle Attack on the full Compression Function of HAS-160 in Encryption Mode

In this section, we give a 71-round related-key rectangle distinguisher, which can be used to mount a related-key rectangle attack on the full 80-round HAS-160 in encryption mode. We can use Property 2 to partially determine whether a candidate quartet is a right one or not. A wrong quartet can be discarded during the stepwise computation, which reduces the complexity of the subsequent steps and also the overall complexity of the attack. Thus, our attack uses the early abort technique.

### 5.1 A 71-Round Related-Key Rectangle Distinguisher

Let  $K$  be a master key which can be written as  $K = x_0, x_1, \dots, x_{15}$ , where  $x_i$  is a 32-bit word. We use four different – but related – master keys  $K^a, K^b, K^c$  and  $K^d$  to mount our related-key rectangle attack on the full HAS-160 encryption mode. The master key differences are as follows:

$$\begin{aligned} \Delta K^* &= K^a \oplus K^b = K^c \oplus K^d = (e_{31}, 0, 0, 0, 0, 0, 0, 0, 0, 0, e_{31}, 0, 0, 0, 0, 0), \quad (1) \\ \Delta K' &= K^a \oplus K^c = K^b \oplus K^d = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, e_{31}, 0). \end{aligned}$$

Since the key schedule of HAS-160 offers a high degree of linearity we can easily determine all the 80 round key differences derived from the master key differences

$\Delta K^*$  and  $\Delta K'$  respectively. We observe that if we choose  $\Delta x_0 = \Delta x_{10}$  and the remaining word differences as zero, i.e.,  $\Delta x_i = 0, i = 1, 2, \dots, 8, 9, 11, 12, \dots, 15$ , then a zero difference can be obtained starting from round 14 up to round 37. We use this observation for the related-key differential for  $E0$ . Moreover, we can observe that if  $\Delta x_{12} = \Delta x_{14}$  holds and the remaining word differences in  $\Delta K'$  are all zero, then a zero difference can be obtained from round 44 to round 65. This observation is used in our related-key differential for  $E1$ .

Considering Property 1 and Theorem 1 we have found a 39-round related-key differential from round 0 to 39 for  $E0$  ( $\alpha \rightarrow \beta$ ) using the master key difference  $\Delta K^*$ . The related-key differential is:

$$(e_7, e_1, 0, e_{5,19,31}, e_{12,26,31}) \rightarrow (e_{4,31}, e_{31}, 0, 0, 0).$$

The related-key differential  $E0$  is shown in Table 4.<sup>3</sup>

**Table 4.** The Related-Key Differential  $E0$

$i$	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta D_i$	$\Delta E_i$	$\Delta k_i$	Prob.
0	$e_7$	$e_1$	0	$e_{5,19,31}$	$e_{12,26,31}$	—	$2^{-7}$
1	$e_{26}$	$e_7$	$e_{11}$	0	$e_{5,19,31}$	$e_{31}$	$2^{-5}$
2	$e_{19}$	$e_{26}$	$e_{17}$	$e_{11}$	0	$e_{31}$	$2^{-6}$
3	0	$e_{19}$	$e_4$	$e_{17}$	$e_{11}$	0	$2^{-5}$
4	$e_{11}$	0	$e_{29}$	$e_4$	$e_{17}$	0	$2^{-3}$
5	$e_{23}$	$e_{11}$	0	$e_{29}$	$e_4$	0	$2^{-4}$
6	$e_{21}$	$e_{23}$	$e_{21}$	0	$e_{29}$	0	$2^{-4}$
7	0	$e_{21}$	$e_1$	$e_{21}$	0	0	$2^{-3}$
8	0	0	$e_{31}$	$e_1$	$e_{21}$	0	$2^{-3}$
9	$e_{21}$	0	0	$e_{31}$	$e_1$	0	$2^{-3}$
10	0	$e_{21}$	0	0	$e_{31}$	0	$2^{-2}$
11	0	0	$e_{31}$	0	0	$e_{31}$	$2^{-1}$
12	0	0	0	$e_{31}$	0	0	$2^{-1}$
13	0	0	0	0	$e_{31}$	0	1
14	0	0	0	0	0	$e_{31}$	1
15	0	0	0	0	0	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
37	0	0	0	0	0	0	1
38	$e_{31}$	0	0	0	0	$e_{31}$	$2^{-1}$
39	$e_{4,31}$	$e_{31}$	0	0	0	0	

<sup>3</sup> Note that  $\Pr[(\Delta c_i, \Delta k_i) \xrightarrow{\boxplus} \Delta k_i] = 1$  always holds due to Property 1. This is true since  $\Delta c_i$  is equal to zero for all  $i$  and  $\Delta k_i$  is either zero or  $e_{31}$ .



We exploit a 32-round related-key differential for  $E1$  ( $\gamma \rightarrow \delta$ ) that covers rounds 39 to 71 using the key difference  $\Delta K'$ . The related-key differential is:

$$(e_6, 0, 0, 0, e_{19}) \rightarrow (e_{5,6,7,14,17,18,19,28,29,30}, e_{5,8,9,19,21,29}, e_{5,26,27}, e_{19}, e_5)$$

The 160-bit difference  $\delta$  can be written as a concatenation of five 32-bit word differences

$$\delta = (\delta_A, \delta_B, \delta_C, \delta_D, \delta_E) = (\Delta A_{71}, \Delta B_{71}, \Delta C_{71}, \Delta D_{71}, \Delta E_{71}). \quad (2)$$

The related-key differential  $E1$  is shown in Table 5. The probability for the

**Table 5.** The Related-Key Differential  $E1$

$i$	$\Delta A_i$	$\Delta B_i$	$\Delta C_i$	$\Delta D_i$	$\Delta E_i$	$\Delta k_i$	Prob.
39	$e_6$	0	0	0	$e_{19}$	—	$2^{-1}$
40	0	$e_6$	0	0	0	0	$2^{-1}$
41	0	0	$e_{31}$	0	0	0	1
42	0	0	0	$e_{31}$	0	$e_{31}$	$2^{-1}$
43	0	0	0	0	$e_{31}$	0	1
44	0	0	0	0	0	$e_{31}$	1
45	0	0	0	0	0	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
65	0	0	0	0	0	0	1
66	$e_{31}$	0	0	0	0	$e_{31}$	$2^{-1}$
67	$e_7$	$e_{31}$	0	0	0	0	$2^{-1}$
68	$e_{21}$	$e_7$	$e_{29}$	0	0	$e_{31}$	$2^{-3}$
69	$e_{7,28,29}$	$e_{21}$	$e_5$	$e_{29}$	0	0	$2^{-6}$
70	$e_{5,8,9,19,21,29}$	$e_{7,28,29}$	$e_{19}$	$e_5$	$e_{29}$	0	$2^{-10}$
71	$e_{5,6,7,14,17,18,19,28,29,30}$	$e_{5,8,9,19,21,29}$	$e_{5,26,27}$	$e_{19}$	$e_5$	0	

differential  $E0$  is  $2^{-48}$  due to Table 4, while the probability for  $E1$  is  $2^{-24}$  from Table 5. Thus, the probability of our related-key rectangle distinguisher for round 1–71 is:

$$(2^{-48} \cdot 2^{-24})^2 \cdot 2^{-160} = 2^{-304}$$

However, the correct difference  $\delta$  occurs in two ciphertext pairs of a quartet for a random cipher with probability  $(2^{-160})^2 = 2^{-320}$ .

## 5.2 The Attack on the full HAS-160 in Encryption Mode

Our attack uses four related keys  $K^a, K^b, K^c$  and  $K^d$  where each two of the four master keys are related as stated in (2). It is assumed that an attacker chooses the

two master key differences  $\Delta K^*$  and  $\Delta K'$ , but not the maser keys themselves. In the first step we apply the 71-round related-key rectangle distinguisher to obtain a small amount of subkey candidates in rounds 72, 73, 74, 76, 77, 78, 80. In the second step we find the remaining subkey candidates by an exhaustive search for the obtained subkey candidates and the remaining subkeys to recover the four 512-bit master keys  $K^a, K^b, K^c$  and  $K^d$ .

The attack works as follows:

1. Choose  $2^{155}$  plaintexts  $P_i^a = (A_i, B_i, C_i, D_i, E_i)$ ,  $i = 1, 2, \dots, 2^{155}$ . Compute  $2^{155}$  plaintexts  $P_i^b$ , i.e.,  $P_i^b = P_i^a \oplus \alpha$ , where  $\alpha$  is a fixed 160-bit word as stated above. Set  $P_i^c = P_i^a$  and  $P_i^d = P_i^b$ . In a chosen plaintext attack scenario ask for the encryption of the plaintexts  $P_i^a, P_i^b, P_i^c, P_i^d$  under  $K^a, K^b, K^c$  and  $K^d$ , respectively, and obtain the ciphertexts  $C_i^a, C_i^b, C_i^c$  and  $C_i^d$ .
2. Guess seven 32-bit round keys  $k_{80}^a, k_{79}^a, k_{78}^a, k_{77}^a, k_{76}^a, k_{75}^a, k_{74}^a$  and compute  $k_{80}^l, k_{79}^l, k_{78}^l, k_{77}^l, k_{76}^l, k_{75}^l, k_{74}^l$ ,  $l \in \{b, c, d\}$  using the known round key differences.
  - 2.1. Decrypt each of the ciphertexts  $C_i^a, C_i^b, C_i^c, C_i^d$  under  $k_{80}^l, k_{79}^l, k_{78}^l, k_{77}^l, k_{76}^l, k_{75}^l, k_{74}^l$ ,  $l \in \{a, b, c, d\}$  respectively and obtain the intermediate encryption values  $C_{73,i}^a, C_{73,i}^b, C_{73,i}^c$  and  $C_{73,i}^d$ , respectively. From Property 2 we know the value of the 96-bit difference  $\delta_{A \ll 30}$ ,  $\delta_{B \ll 30}$  and  $\delta_C$ , see (2).
  - 2.2. Check whether the following conditions are fulfilled for any quartet  $(C_{73,j}^a, C_{73,j}^b, C_{73,j}^c, C_{73,j}^d)$ :

$$\begin{aligned} C_{73,j}^a \oplus C_{73,j}^c &= \delta_{A \ll 30} = C_{73,j}^b \oplus C_{73,j}^d, \\ D_{73,j}^a \oplus D_{73,j}^c &= \delta_{B \ll 30} = D_{73,j}^b \oplus D_{73,j}^d, \\ E_{73,j}^a \oplus E_{73,j}^c &= \delta_C = E_{73,j}^b \oplus E_{73,j}^d \end{aligned}$$

Discard all the quartets that do not satisfy the above conditions and discard the quartets that do not satisfy this condition.

3. Guess a 32-bit round key  $k_{73}^a$  and compute  $k_{73}^l$ ,  $l \in \{b, c, d\}$  using the known round key differences.
  - 3.1. Decrypt each remaining quartet  $(C_{73,j}^a, C_{73,j}^b, C_{73,j}^c, C_{73,j}^d)$  under  $k_{73}^l$ ,  $l \in \{a, b, c, d\}$ , respectively and obtain the quartets  $(C_{72,j}^a, C_{72,j}^b, C_{72,j}^c, C_{72,j}^d)$ .
  - 3.2. Check whether  $E_{72,j}^a \oplus E_{72,j}^c = \delta_D = E_{72,j}^b \oplus E_{72,j}^d$  holds. From Property 2 we know the value of the 32-bit difference  $\delta_D$ . Discard all the quartets that do not satisfy the above condition.
4. Guess one 32-bit round keys  $k_{72}^a$  and compute  $k_{72}^l$ ,  $l \in \{b, c, d\}$  using the known round key differences.
  - 4.1. Decrypt each remaining quartet  $(C_{72,j}^a, C_{72,j}^b, C_{72,j}^c, C_{72,j}^d)$  under  $k_{72}^l$ ,  $l \in \{a, b, c, d\}$  respectively and obtain the quartets  $(C_{71,j}^a, C_{71,j}^b, C_{71,j}^c, C_{71,j}^d)$ .
  - 4.2. Check whether  $E_{71,j}^a \oplus E_{71,j}^c = \delta_E = E_{71,j}^b \oplus E_{71,j}^d$  holds. From Property 2 we know the value of the 32-bit difference  $\delta_E$ . If there exist at least 21 quartets passing the above condition, record  $k_{80}^l, k_{79}^l, k_{78}^l, k_{77}^l, k_{76}^l, k_{75}^l, k_{74}^l, k_{73}^l, k_{72}^l$ ,  $l \in \{a, b, c, d\}$  and go to Step 5. Otherwise go to Step 4 with another guessed round key. If all the possible round keys for  $k_{72}^a$  are tested, then repeat Step

- 3 with another guessed round key  $k_{73}^a$ . If all the possible round keys for  $k_{73}^a$  are tested, then go to Step 2 with another guess for the round keys  $k_{80}^a, k_{79}^a, k_{78}^a, k_{77}^a, k_{76}^a, k_{75}^a, k_{74}^a$ .
5. For a suggested  $(k_{80}^l, k_{79}^l, k_{78}^l, k_{77}^l, k_{76}^l, k_{75}^l, k_{74}^l, k_{73}^l, k_{72}^l)$ , do an exhaustive key search for the remaining  $512 - 9 \cdot 32 = 224$  key bits by trial encryptions. If a 512-bit key is suggested, output it as the master key of the full HAS-160 encryption mode. Otherwise, restart the algorithm.

### 5.3 Analysis of the Attack

There are  $2^{155}$  pairs  $(P_i^a, P_i^b)$  and  $2^{155}$  pairs  $(P_i^c, P_i^d)$  of plaintexts, thus we have  $(2^{155})^2 = 2^{310}$  quartets. The data complexity of Step 1 is  $2^2 \cdot 2^{155} = 2^{157}$  chosen plaintexts. The time complexity of Step 1 is about  $2^2 \cdot 2^{155} = 2^{157}$  encryptions. Step 2.1 requires time about  $2^{224} \cdot 2^2 \cdot 2^{155} \cdot (7/80) \approx 2^{377.5}$  eighty round encryptions. The number of remaining quartets after Step 2.2 is  $2^{310} \cdot (2^{-96})^2 = 2^{118}$ , since we have a 96-bit filtering condition on both pairs of a quartet. The time complexity of Step 3.1 is about  $2^{256} \cdot 2^2 \cdot 2^{118} \cdot (1/80) \approx 2^{370}$  encryptions. After Step 3.2 about  $2^{118} \cdot (2^{-32})^2 = 2^{54}$  quartets remain, since we have a 32-bit filtering condition on both pairs of a quartet. The time complexity of Step 4.1 is  $2^{288} \cdot 2^2 \cdot 2^{54} \cdot (1/80) \approx 2^{337.5}$  encryptions. After Step 4.2 the number of remaining quartets is about  $2^{54} \cdot (2^{-32})^2 = 2^{-10}$  for each subkey guess, since we have a 32-bit filtering condition on both pairs of a quartet. The expected number of quartets that remain for the correct round keys are about  $2^{310} \cdot 2^{-304} = 2^6$ .

Using the Poisson distribution we can compute the success rate of our attack. The probability that the number of remaining quartets for each false key bit combination is larger than 21 is  $Y_i \sim \text{Poisson}(\mu = 2^{-10})$ ,  $\Pr(Y_i \geq 22) \approx e^{-2^{-10}} \cdot \frac{(2^{-10})^{22}}{22!}$ , where  $i$  indicates a wrong key. Thus, for all the  $2^{288} - 1$  wrong keys we expect that about  $2^{188} \cdot 2^{-189} = 2^{-1}$  quartets are counted. The probability that the number of quartets counted for the correct key bits is at least 21 is  $Z \sim \text{Poisson}(\mu = 2^6)$ ,  $\Pr(Z \geq 22) \approx 1$ . The data complexity of our attack is  $2^{155} \cdot 2^2 = 2^{157}$  chosen plaintexts, while the time complexity is about  $2^{377.5}$  full HAS-160 encryptions. Our attack has a success rate of 1.

## 6 Conclusion

In this paper we present the first cryptanalytic result on the inner block cipher of the Korean hash algorithm standard HAS-160. Our related-key rectangle attack can break the full block cipher. A more complex and non-linear key schedule would have defended our attack. Moreover, to strengthen the cipher against differential attacks, we propose to use the  $f$ -function more often in each round and so the  $f$ -function may influence more than one word in each round. Note that this analysis does not seem to say anything about the collision, preimage, or 2nd-preimage resistance of HAS-160, but it shows some interesting properties that occur if HAS-160 is used as a block cipher.

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