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Using Color Correlation To Improve Restoration Of Color Images

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1 Abstract

The problem addressed in this work is restoration of images that have a few channels of information. We have studied color images so far, but hopefully the ideas presented here apply to other types of images with more than one channel.

The suggested method is to use a probabilistic scheme which proved rather useful for image restoration, and incorporate into it an additional term, which results in a better correlation between the three color bands in the restored image. Initial results are good; typically, there's a reduction of 30% in the RMS error, compared to standard restoration carried out separately on each color band.

2 Introduction

A rather general formulation of the restoration problem is the following: Given some partial information D on a image F, find the best restoration for F. Obviously, there are many possible ways in which to define "best". One way, which proved quite successful for a wide variety of applications, is probabilistic in nature: Given D, one seeks the restoration $\hat{F}$ which maximizes the probability $Pr(F/D)$. Following Bayes' rule, this is equal to $\frac{Pr(D/F)Pr(F)}{Pr(D)}$. The denominator is a constant once D is measured; $Pr(D/F)$ is usually easy to compute. $Pr(F)$ is more interesting, and more difficult to define. Good results have been obtained by following the physical model of the Boltzman distribution, according to which the probability of a physical system to be in a certain state is proportional to the exponent of the negative of the energy of that state – that is, low-energy, or "ordered" states, are assigned higher probability than high-energy, or "disordered", states [3, 7]. It is common to define the energy of a sign al by its "smoothness"; the energy of a one-dimensional signal F is often defined by $\int F_{xx}^2 dx$, etc. Such integrals are usually called "smoothing terms", as they enforce the resulting restoration to be smooth [5, 8, 4, 6]. Note that here "smooth" does not mean "infinitely differentiable", but "slowly changing".

3 Main Body

To see how the probabilistic approach naturally leads to restoration by so-called "smoothing", or regularization, let us look at the problem of reconstructing a two-dimensional image from sparse samples, which are corrupted by additive noise. Suppose the image is sampled at the points $\{x_i, y_i\}$, the sample values are z_i, and the measurement noise is Gaussian with variance σ^2. Then

$$Pr(D/F) \propto \exp(-\sum_{i=1}^{n} \frac{[F(x_i, y_i) - z_i]^2}{2\sigma^2})$$

and, based on the idea of the Boltzman distribution, one can define $Pr(F)$ as being proportional to

$$\exp(-\lambda \int\int (F_{uu}^2 + 2F_{uv}^2 + F_{vv}^2) du dv)$$

so, the overall probability to maximize is

$$\exp(-(\sum_{i=1}^{n} \frac{[F(x_i, y_i) - z_i]^2}{2\sigma^2} + \lambda \int\int (F_{uu}^2 + 2F_{uv}^2 + F_{vv}^2) du dv))$$

which is, of course, equivalent to minimizing

$$\sum_{i=1}^{n} \frac{[F(x_i, y_i) - z_i]^2}{2\sigma^2} + \lambda \int\int (F_{uu}^2 + 2F_{uv}^2 + F_{vv}^2) du dv \tag{1}$$

This leads, via calculus of variations, to a partial differential equation, which can be effectively solved using multigrid methods. Other problems – such as deblurring – can be posed

First, let us look at the problem of deblurring a single-channel image (for instance, a gray level image). One is given a gray-level image D, which is a corrupted version of the true image F, and the goal is to reconstruct this F. Typically, one assumes that F was blurred by convolution with a kernel H, and corrupted by additive noise, which results in the mathematical model $D = F * H + N$, where $*$ stands for the convolution operator and N is additive noise. Proceeding as in the paradigm described above, one searches for the $\hat{F}$ which minimizes

$$\|D - F * H\|^2 + \lambda \int\int (F_{xx}^2 + 2F_{xy}^2 + F_{yy}^2) dxdy$$

let us proceed to shortly describe how this idea is extended to restoring multi-channeled images.

Now, suppose we are given a color image, with RGB channels, that underwent degradation by convolution with H (for simplicity's sake, assume it is the same H for all channels, although it doesn't have to be so in the general case). One obvious way to reconstruct the image is to run the deblurring algorithm described above, for each of the separate channels, and combine the restored channels into a color image. Such an approach, however, does not work well in general. Usually, the resulting image is still quite blurry, and contaminated by false colors; that is, certain areas contain streaks of colors which do not exist in the original image. This problem is more acute in highly textured areas.

The proposed solution to these problems is to incorporate into the probabilistic scheme a "correlation term", which will result in a better correlation between the RGB channels. Formally, if $C_{x,y}$ is the covariance matrix of the RGB values at a pixel (x, y), the probability for the combination of colors $(R(x,y)\ G(x,y)\ B(x,y))$ is proportional to
$\exp(-\frac{1}{2}(R(x,y)\ G(x,y)\ B(x,y))C_{x,y}^{-1}(R(x,y)\ G(x,y)\ B(x,y)))$. Multiplying over all the pixels results in adding these terms in the exponent's power. Exactly as in the interpolation problem above, this exponential term combines with the other exponential terms, and we get a combined exponential that has to be maximized; therefore, we have to minimize the negative of the power, which simply results in adding the "correlation term", $\int\int (R(x,y)\ G(x,y)\ B(x,y))C_{x,y}^{-1}(R(x,y)\ G(x,y)\ B(x,y))^t dxdy$, to the expression of Eq. 1 (after subtracting the averages of the RGB channels). In effect, this term makes use of the fact that, in natural and synthetic images, the RGB channels are usually highly correlated. The "correlation term" penalizes deviations from this correlation, thus "pushing" the restored image towards one whose channels are "correctly correlated".

Therefore, the combined expression to minimize is

$$\|D - F * H\|^2 + \lambda_1 \Big(\int\int (R_{xx}^2 + 2R_{xy}^2 + R_{yy}^2) dxdy + \int\int (G_{xx}^2 + 2G_{xy}^2 + G_{yy}^2) dxdy + \int\int (B_{xx}^2 + 2B_{xy}^2 + B_{yy}^2) dxdy\Big) + \lambda_2 \int\int (R(x,y)\ G(x,y)\ B(x,y))C_{x,y}^{-1}(R(x,y)\ G(x,y)\ B(x,y))^t dxdy \quad (2)$$

We have implemented a simple iterative scheme for minimizing this functional. A substantial improvement was obtained using the "correlation term". A color photograph was blurred, and restored with and without the correlation term. When using this term, the resulting restoration is sharper, and contains less "false colors". Comparing it against the original image shows that the RMS error is about 30% smaller than when restoring each channel separately.

We have also used the "correlation term" to solve the "demosaicing" problem, in which one has to reconstruct a color image, given only one color band at each pixel [1, 2]. This was accomplished by incorporating the "correlation term" into the solution to the interpolation problem described above; usually, this also resulted in a reduction of about 30% in the RMS error.

4 Summary

An algorithm was suggested to restoring multi-channel images; it uses the correlation between the different channels to improve results. The algorithm was applied to color images and it usually resulted in an improvement of 30% or so in the RMS error as compared to standard restoration applied separately to each channel.

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